

Learning Equilibria of Simulation-Based Games: Applications to Empirical Mechanism Design

Enrique Areyan Viqueira
June 1, 2020

Advisor: Dr. Amy Greenwald



Outline

- **Part 1:** Simulation-Based Games
- **Part 2:** Empirical Mechanism Design
- **Part 3:** Proposed Work

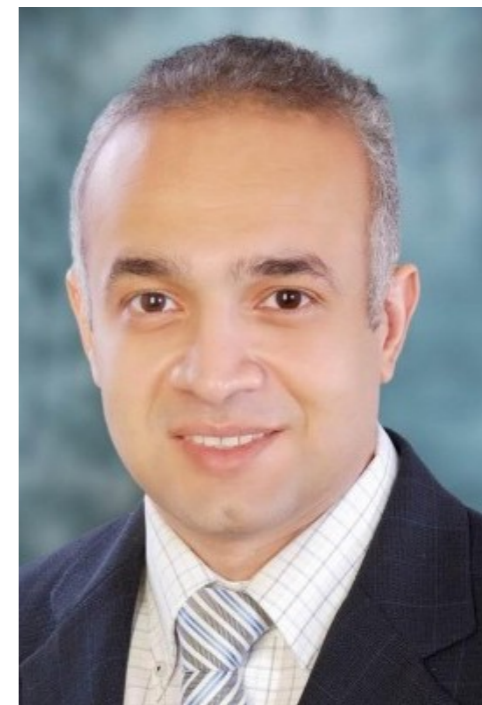
Collaborators



Amy Greenwald



Cyrus Cousins



Yasser Mohammad

(Tentative) Thesis Statement

Thesis Statement

Through modern statistical tools, sampling heuristics, and optimization techniques, we find sample-efficient algorithms that learn the approximate equilibria of simulation-based games and use them to empirically design mechanisms.

Part 1:

Learning Equilibria of Simulation-Based Games

Improved Algorithms for Learning Equilibria in Simulation-Based Games.

Enrique Areyan Viqueira, Cyrus Cousins, Amy Greenwald.

19th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS20).

Learning Simulation-Based Games from Data.

Enrique Areyan Viqueira, Amy Greenwald, Cyrus Cousins, Eli Upfal.

18th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS19).

The "Game" Plan (a.k.a. Outline Part 1)

- Simulation-based Games
- Mathematical Framework
- Learning Algorithms
- Experimental Results

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- **Game theory** is the standard conceptual framework to analyze the interaction among strategic agents

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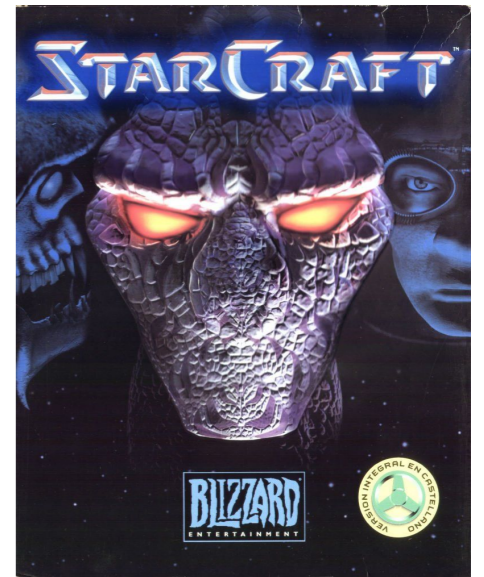
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- **Game theory** is the standard conceptual framework to analyze the interaction among strategic agents
- At the heart of game theory is the notion of a **Game** - a mathematical object: players, actions, and utilities
- Often, an analyst can specify a **game description** completely. But, there are games too complex to afford a complete description

Simulation-Based Games - Examples

- StarCraft: a real-time strategy game
- Hundreds of units and buildings, large strategy space
- Deepmind¹ recently built the first AI to defeat a top player
Their parameterization of the game has an average of 10^{26} legal actions at each step!



[1] <https://deepmind.com/blog/article/alphastar-mastering-real-time-strategy-game-starcraft-ii>

Simulation-Based Games - Pervasive in Real Life

- As fun as StarCraft might be, think of it as a model for important, real-world applications such as:

Electronic advertisement (TAC AdX - <https://sites.google.com/site/gameadx/>)

Energy markets (Power TAC - <https://powertac.org/>)

Industrial supply chains (ANAC-SCML <http://web.tuat.ac.jp/~katfuji/ANAC2019/#scm>)

etc.

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- Many sources of complexity, in the StarCraft example different terrains, units, actions, etc.
- Nevertheless, in **simulation-based games**, one can obtain samples of utilities by running a **game simulator**

Simulation-Based Games - Mathematical Model

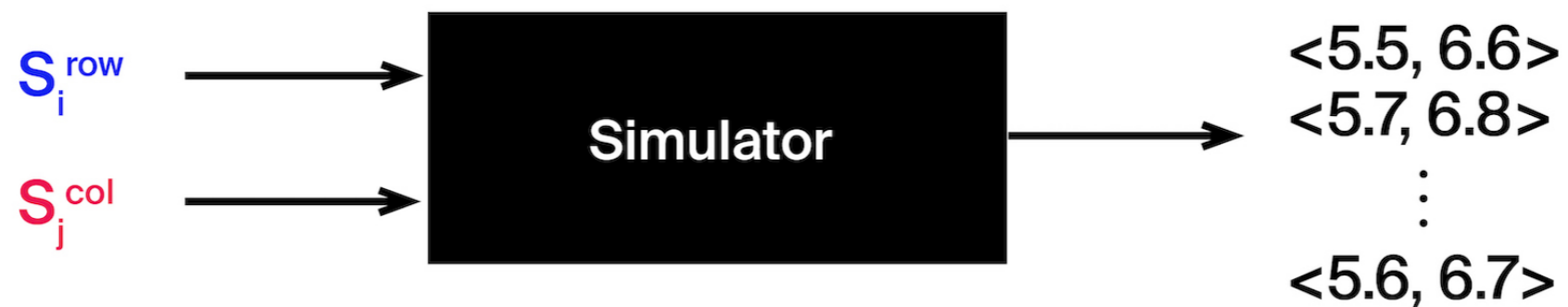
	S_1^{col}	S_2^{col}	\dots	S_n^{col}
S_1^{row}	?, ?	?, ?	\dots	?, ?
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Simulation-based game

Simulation-Based Games - Mathematical Model

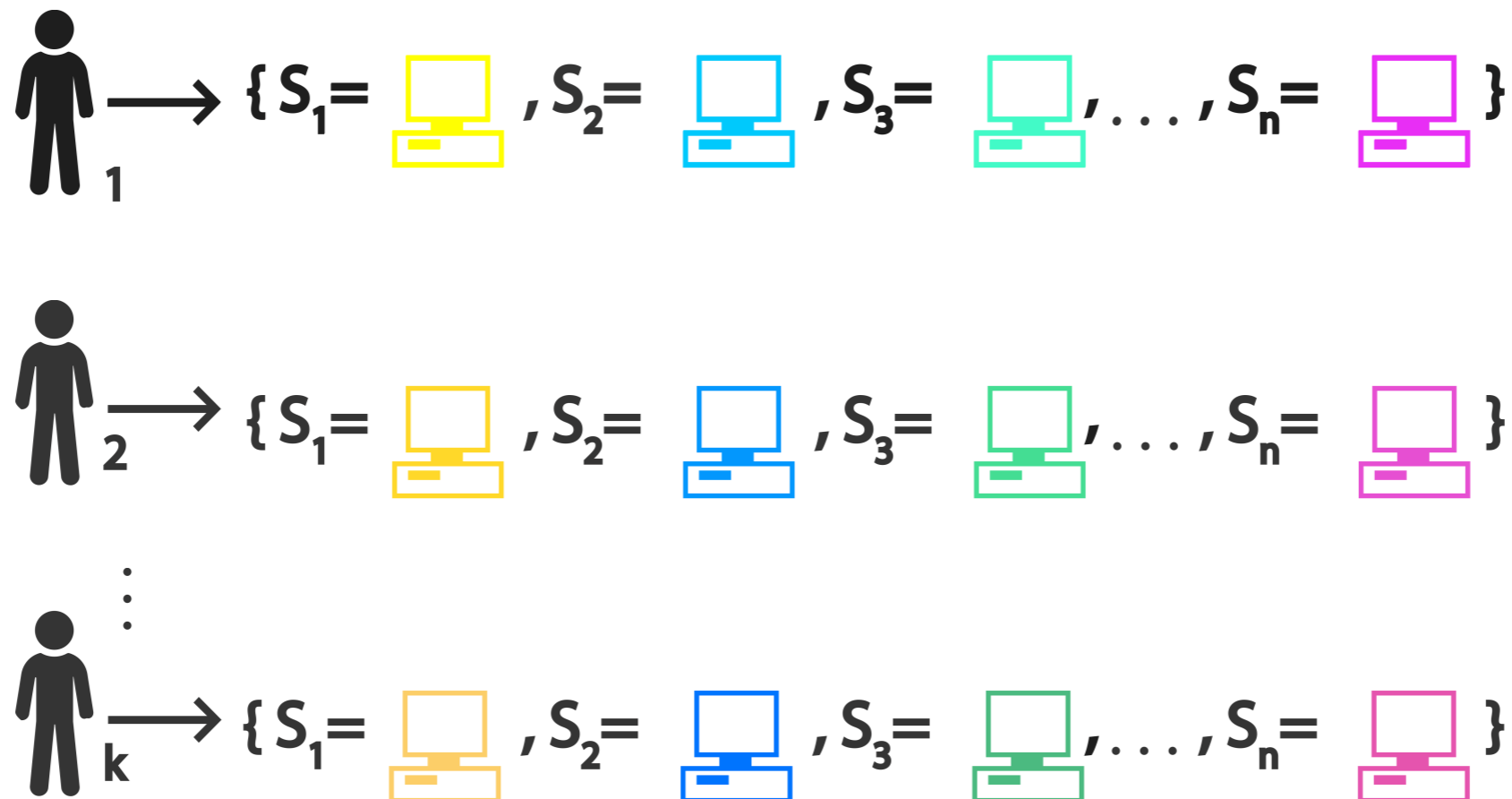
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Simulation-based game



Simulation-Based Games - Heuristics

- Actions spaces are vast, so usually no optimal strategies are available. Instead, there are a few heuristics.



Plan for the rest of Part 1

- High-level Goal: learn the equilibria of simulation-based games
- Formalize simulation-based games and their equilibria
- Learning algorithms and experimental results

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A Mathematical Model - Conditional and Expected Games

$\vec{s} = (s_1, s_2, \dots, s_n)$, where s_i is agent's i strategy

- Let $u_p(\vec{s})$ be player p 's utility when **strategy profile** \vec{s} is played

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- The **expected game** (the normal-form game with expected utilities) is then our model of a simulation-based game

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The **empirical utility** is the average: $\hat{u}_p(\vec{s}) = \frac{1}{m} \sum_{i=1}^m u_p(\vec{s}; x_i)$

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- The **empirical game** has empirical utilities for every player and strategy profile

Goal

Learn, with provable guarantees, **all** the **equilibria** of **expected games** given access only to **empirical games**

(Other valid and interesting goals:

- + recover one equilibrium, e.g., by following best-response dynamics)

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G_2	s_1^{col}	s_2^{col}
s_1^{row}	-1, -1	-3, $0 + \epsilon$
s_2^{row}	$0, -3 - \epsilon$	$-2 + \frac{\epsilon}{4}, -2$

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G_1 is ϵ -close to G_2



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- For simulation-based games, \mathcal{X} , and \mathcal{D} are complex objects. We can't reasonably hope to compute the expected game exactly
- Even if we could approximate each $\bar{u}_p(\vec{s})$ (say, up to ε), would that destroy the equilibria?
- **Definition:** a strategy profile \vec{s} is an ε -**equilibrium** if players don't have incentive to deviate, up to ε , fixing other players' strategies

Approximating Equilibria - First Result

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Tuyls, K. et al.
Bounds and dynamics
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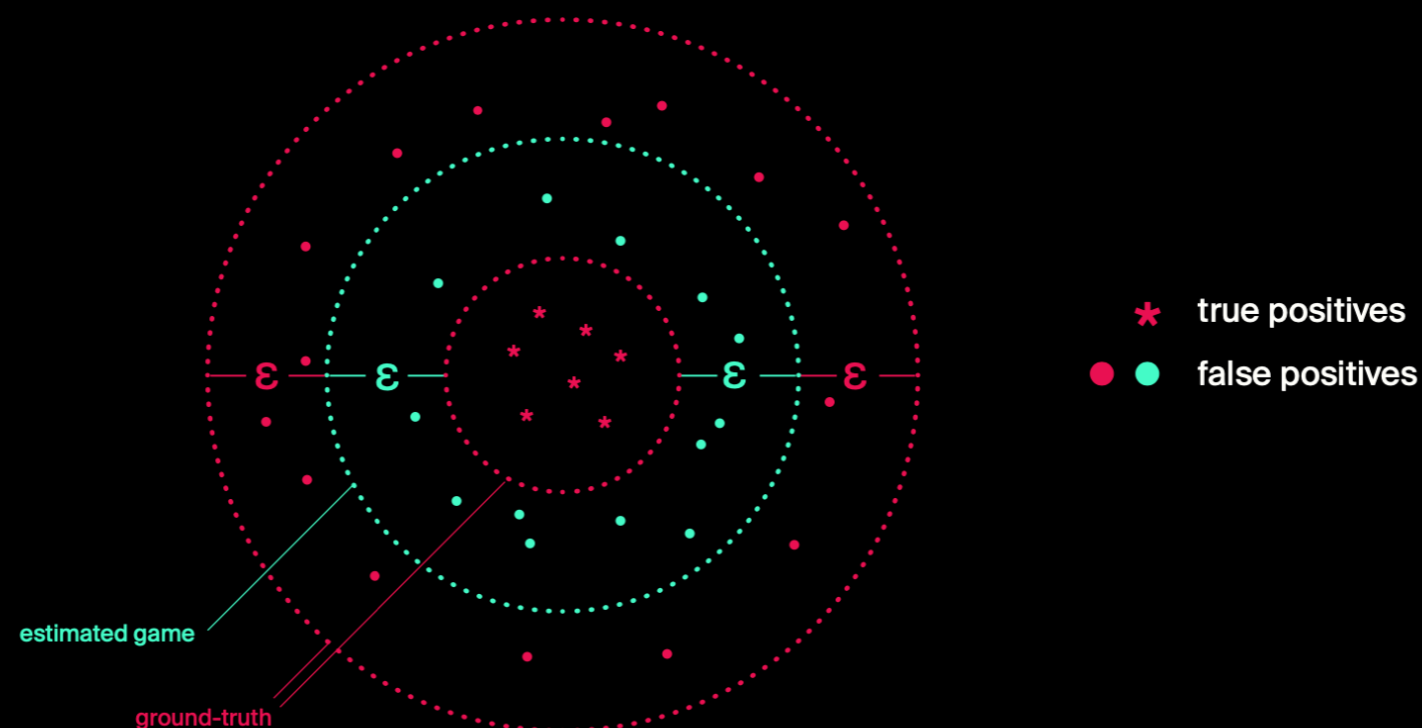
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With probability at least $1-\delta$

$$N(\text{ground-truth game}) \subseteq N_{2\epsilon}(\text{estimated game}) \subseteq N_{4\epsilon}(\text{ground-truth game})$$



Learning Equilibria

How to learn the approximate equilibria
of a simulation-based game from sample data?

Original
Goal



How to learn an ϵ -uniform approximation of
an expected game from sample data?

Mathematically
Precise Goal

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Learning Algorithms - A Baseline

- We present two **Probably Approximate Correct** (PAC) algorithm to learn empirical games
 - **PAC** algorithm: given $\varepsilon, \delta > 0$, learn some model (games!) up to error at most ε and with probability at least $1 - \delta$
- The first algorithm is a baseline that uses **Hoeffding's Inequality** to estimate all utilities of a simulation-based game

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s_1^{row}	3, 3	0, 5
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What value of ε is enough to estimate the equilibrium of this game?

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We don't need to learn "3" exactly, we just need to learn that "5" > "3", up to errors.

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- **Idea:** take a few samples first, then take more samples of only those profiles that can't be refuted as part of an equilibrium

Learning Algorithms - Progressive Sampling with Pruning

- **Algorithm:** Progressive Sampling With Pruning

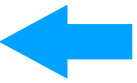
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- Initially, all p, \vec{s} are active. Initial error ϵ_0 is "big".
- While some target accuracy ϵ is not reached ($\epsilon < \epsilon_t$) or  or
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 - if \vec{s} can be refuted as part of an equilibrium, then remove it from the active set



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 - For all active \vec{s}
 - if \vec{s} can be refuted as part of an equilibrium, then remove it from the active set
 - Decrease the target error $\epsilon_{t+1} \leftarrow \epsilon_t - \text{constant}$



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Experimental Setup

- We use GAMUT (gamut.stanford.edu) to generate games
- We use Gambit (www.gambit-project.org) for equilibria computation
- We developed a python library (github.com/eareyan/pysegta) that implements our learning algorithms and interfaces with both GAMUT and Gambit.

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- Pruning is highly data efficient over a wide range of games, specifically, over 10 different classes of games
- Efficiency is due to the algorithm exploiting the strategic structure of games without knowing *a priori* what this structure is!
- In our paper, we also discussed a rather pathological example of a game where pruning is not effective

Experimental Results - Summary

Bound	$\varepsilon \leq 0.125$		$\varepsilon \leq 0.25$		$\varepsilon \leq 0.5$		$\varepsilon \leq 1.0$	
	Hoeffding	Emp. Bennett	Hoeffding	Emp. Bennett	Hoeffding	Emp. Bennett	Hoeffding	Emp. Bennett
Game/Algorithm	GS; PSP; ε_{PSP}	GS; PSP; ε_{PSP}	GS; PSP; ε_{PSP}	GS; PSP; ε_{PSP}	GS; PSP; ε_{PSP}	GS; PSP; ε_{PSP}	GS; PSP; ε_{PSP}	GS; PSP; ε_{PSP}
Congestion Games (5 facilities)	3,051; 1,654 ; 0.08	3,051; 1,449 ; 0.00	762; 464 ; 0.17	762; 364 ; 0.01	190; 146 ; 0.34	190; 93 ; 0.01	47 ; 58; 0.70	47; 25 ; 0.04
Zero-Sum Games (30 strategies)	2,841; 1,691 ; 0.08	2,841; 1,383 ; 0.00	710; 502 ; 0.17	710; 349 ; 0.01	177; 166 ; 0.35	177; 90 ; 0.01	44 ; 62; 0.71	44; 25 ; 0.04
Random Games (30 strategies)	2,841; 1,666 ; 0.08	2,841; 1,375 ; 0.00	710; 491 ; 0.17	710; 347 ; 0.01	177; 159 ; 0.35	177; 90 ; 0.01	44 ; 58; 0.71	44; 25 ; 0.04
Congestion Games (4 facilities)	622; 492 ; 0.09	622; 438 ; 0.00	156; 138 ; 0.17	156; 110 ; 0.01	39 ; 41; 0.35	39; 28 ; 0.01	10 ; 15; 0.71	10; 8 ; 0.04
Zero-Sum Games (20 strategies)	1,171; 829 ; 0.09	1,171; 708 ; 0.00	293; 240 ; 0.17	293; 179 ; 0.01	73 ; 77; 0.35	73; 46 ; 0.01	18 ; 28; 0.71	18; 13 ; 0.04
Random Games (20 strategies)	1,171; 809 ; 0.09	1,171; 698 ; 0.00	293; 232 ; 0.17	293; 176 ; 0.01	73 ; 73 ; 0.35	73; 45 ; 0.01	18 ; 25; 0.71	18; 12 ; 0.04
Congestion Games (3 facilities)	114 ; 145; 0.09	114 ; 135; 0.00	29 ; 40; 0.18	29 ; 34; 0.01	7; 12; 0.36	7; 9; 0.02	2 ; 4; 0.73	2 ; 2 ; 0.05
Zero-Sum Games (10 strategies)	254 ; 268; 0.09	254; 242 ; 0.00	63 ; 73; 0.18	63; 61 ; 0.01	16 ; 22; 0.36	16; 15 ; 0.02	4 ; 7; 0.73	4 ; 4 ; 0.05
Random Games (10 strategies)	254 ; 254 ; 0.09	254; 233 ; 0.00	63 ; 69; 0.18	63; 59 ; 0.01	16 ; 21; 0.36	16; 15 ; 0.02	4 ; 7; 0.72	4 ; 4 ; 0.05
Congestion Games (2 facilities)	17 ; 37; 0.09	17 ; 37; 0.00	4 ; 10; 0.19	4 ; 9; 0.01	1 ; 3; 0.38	1 ; 2; 0.02	1 ; 1 ; 0.76	1 ; 1 ; 0.05
Zero-Sum Games (5 strategies)	54 ; 94; 0.09	54 ; 89; 0.00	13 ; 25; 0.18	13 ; 22; 0.01	3 ; 7; 0.37	3 ; 6; 0.02	1 ; 2; 0.75	1 ; 1 ; 0.05
Random Games (5 strategies)	54 ; 83; 0.09	54 ; 90; 0.00	13 ; 22; 0.18	13 ; 20; 0.01	3 ; 6; 0.37	3 ; 5; 0.02	1 ; 2; 0.74	1 ; 1 ; 0.05

Table 1: PSP's sample efficiency. Numbers of samples are reported in tens of thousands. The values in bold are smaller than their counterparts; as ε is fixed, they indicate the more sample efficient algorithms.

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Zero-Sum Games (20 strategies)	1,171; 829 ; 0.09	1,171; 708 ; 0.00	293; 240 ; 0.17	293; 179 ; 0.01	73 ; 77; 0.35	73; 46 ; 0.01	18 ; 28; 0.71	18; 13 ; 0.04
Random Games (20 strategies)	1,171; 809 ; 0.09	1,171; 698 ; 0.00	293; 232 ; 0.17	293; 176 ; 0.01	73 ; 73 ; 0.35	73; 45 ; 0.01	18 ; 25; 0.71	18; 12 ; 0.04
Congestion Games (3 facilities)	114 ; 145; 0.09	114 ; 135; 0.00	29 ; 40; 0.18	29 ; 34; 0.01	7; 12; 0.36	7; 9; 0.02	2 ; 4; 0.73	2 ; 2 ; 0.05
Zero-Sum Games (10 strategies)	254 ; 268; 0.09	254; 242 ; 0.00	63 ; 73; 0.18	63; 61 ; 0.01	16 ; 22; 0.36	16; 15 ; 0.02	4 ; 7; 0.73	4 ; 4 ; 0.05
Random Games (10 strategies)	254 ; 254 ; 0.09	254; 233 ; 0.00	63 ; 69; 0.18	63; 59 ; 0.01	16 ; 21; 0.36	16; 15 ; 0.02	4 ; 7; 0.72	4 ; 4 ; 0.05
Congestion Games (2 facilities)	17 ; 37; 0.09	17 ; 37; 0.00	4 ; 10; 0.19	4 ; 9; 0.01	1 ; 3; 0.38	1 ; 2; 0.02	1 ; 1 ; 0.76	1 ; 1 ; 0.05
Zero-Sum Games (5 strategies)	54 ; 94; 0.09	54 ; 89; 0.00	13 ; 25; 0.18	13 ; 22; 0.01	3 ; 7; 0.37	3 ; 6; 0.02	1 ; 2; 0.75	1 ; 1 ; 0.05
Random Games (5 strategies)	54 ; 83; 0.09	54 ; 90; 0.00	13 ; 22; 0.18	13 ; 20; 0.01	3 ; 6; 0.37	3 ; 5; 0.02	1 ; 2; 0.74	1 ; 1 ; 0.05

Table 1: PSP's sample efficiency. Numbers of samples are reported in tens of thousands. The values in bold are smaller than their counterparts; as ε is fixed, they indicate the more sample efficient algorithms.

Experimental Results - Summary

Bound	$\epsilon \leq 0.125$	
	Hoeffding	Emp. Bennett
Game/Algorithm	GS; PSP; ϵ_{PSP}	GS; PSP; ϵ_{PSP}
Congestion Games (5 facilities)	3,051; 1,654 ; 0.08	3,051; 1,449 ; 0.00
Zero-Sum Games (30 strategies)	2,841; 1,691 ; 0.08	2,841; 1,383 ; 0.00
Random Games (30 strategies)	2,841; 1,666 ; 0.08	2,841; 1,375 ; 0.00
Congestion Games (4 facilities)	622; 492 ; 0.09	622; 438 ; 0.00
Zero-Sum Games (20 strategies)	1,171; 829 ; 0.09	1,171; 708 ; 0.00
Random Games (20 strategies)	1,171; 809 ; 0.09	1,171; 698 ; 0.00
Congestion Games (3 facilities)	114 ; 145; 0.09	114 ; 135; 0.00
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Random Games (5 strategies)	54 ; 83; 0.09	54 ; 90; 0.00

0
mp. Bennett
GS; PSP; ϵ_{PSP}
47; 25; 0.04
44; 25; 0.04
44; 25; 0.04
10; 8; 0.04
18; 13; 0.04
18; 12; 0.04
2; 2; 0.05
4; 4; 0.05
4; 4; 0.05
1; 1; 0.05
1; 1; 0.05
1; 1; 0.05

smaller than

The "Game" Plan (a.k.a. Outline Part 1)

- ~~Simulation-based Games~~
- ~~Mathematical Framework~~
- ~~Learning Algorithms~~
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Summary Part 1

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- We contribute an end-to-end methodology for the analysis of simulation-based games
- We prove tight bounds on the set of approximate equilibria of games learned from data
- We present and empirically evaluate a learning algorithm that exploits strategic structure of games to save on samples
- We contribute an open-source library that implements our learning algorithms www.github.com/eareyan/pysegta

Part 2:

Empirical Mechanism Design

Empirical Mechanism Design: Designing Mechanisms from Data.

Enrique Areyan Viqueira, Cyrus Cousins, Yasser Mohammad, Amy Greenwald.
Uncertainty in Artificial Intelligence (UAI19).

On Approximate Welfare-and Revenue-Maximizing Equilibria for Size-Interchangeable Bidders.

Enrique Areyan Viqueira, Amy Greenwald, Victor Naroditskiy.
16th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS17).

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Mechanism design: designing games so that the ensuing behavior of agents, at equilibrium, leads to desirable outcomes.

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Examples abound:

- Design of auctions
- Designing negotiation protocols
- Design of college admission systems
- etc.



The Rules of the Game Matter

Bangladesh

Bangladesh raises USD1.7bn from LTE frequency tender

15 Feb 2018

 Bangladesh

The Bangladeshi government has raised a total of BDT52.89 billion (USD1.68 billion) from its 4G spectrum auction, far below the expected BDT110 billion figure, with less than 30% of the 46.4MHz of spectrum put up for sale bought in the tender, The Daily Star writes. Shahjahan Mahmood, chairman of the BTRC, said the regulator was 'not happy' with the results of the auction, adding that the operators will have another opportunity to acquire spectrum at the same price within the next six months.

Market leader GrameenPhone will pay USD408 billion for 5MHz in the 1800MHz band, in addition to a fee to convert its current holdings in the 900MHz and 1800MHz bands so as to make it technology neutral. Banglalink was awarded 2x5.6MHz in the 1800MHz band and 5MHz of paired spectrum in the 2100MHz band for a total fee of USD308.6 million (excluding VAT), while it will pay a further USD35 million to convert its existing spectrum

"Bangladesh raises USD1.7bn from LTE frequency tender." 15 Feb. 2018, [https://www.telegeography.com/products/commupdate/articles/..](https://www.telegeography.com/products/commupdate/articles/)

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Airwaves in the 2,300 megahertz band sold out as telecom operators spent to increase their 4G mobile broadband services. Photo: Mint

Spectrum auction ends, govt makes Rs65,789 crore, misses target

4 min read . Updated: 07 Oct 2016, 10:08 AM IST

Upasana Jain

Proceeds from spectrum auction a fraction of the Rs5.63 trillion of
airwaves on offer; no bids were received for 700 MHz, 900 Mhz bands

Bangladesh

India

"Spectrum auction ends, govt makes
Rs65,789 crore, misses target."
07 Oct. 2016, [https://www.livemint.com/
Industry/xt5r4Zs5RmzjdWuLUdwJMI/](https://www.livemint.com/Industry/xt5r4Zs5RmzjdWuLUdwJMI/)..

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India

MTN Ghana poised to snap up unallocated 800MHz 4G spectrum

Ghana

5 Apr 2019

 Ghana

Mobile network operator (MNO) MTN Ghana is lining up to purchase the two remaining 2x5MHz blocks of spectrum lots in the 800MHz band that were left unallocated after Vodafone Ghana acquired its own block of 2x5MHz for USD30 million last December, Adom News reports. 'MTN intends to acquire this remaining spectrum to enable it to continue to give its customers an increasingly better experience on the network,' MTN Corporate Services Executive Robert Kuzoe confirmed to Adom News in response to a questionnaire.

The MNO was precluded from the National Communications Authority (NCA's) auction of three separate 2x5MHz spectrum lots in the 800MHz band at the end of last year, on the grounds that it had already acquired a 2x10MHz lot in the same band back in December 2015. While the NCA confirmed at the end of the 2018 spectrum auction that 'two companies submitted applications, with Vodafone emerging as the only successful applicant,' the

"MTN Ghana poised to snap up unallocated 800MHz 4G spectrum."
05 April. 2019, [https://www.telegeography.com/products/commsupdate/..](https://www.telegeography.com/products/commsupdate/)

Empirical Mechanism Design

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How should a **mechanism designer** set **parameters** of a mechanism, given access only to **data** (or to a simulator capable of generating data) about the **game** under different choices of parameters?

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How should a **mechanism designer** set **parameters** of a mechanism, given access only to **data** (or to a simulator capable of generating data) about the **game** under different choices of parameters?

e.g., How should an **auctioneer** set the **reserve prices** of an auction given access only to auction log **data under different choices of reserve prices**?

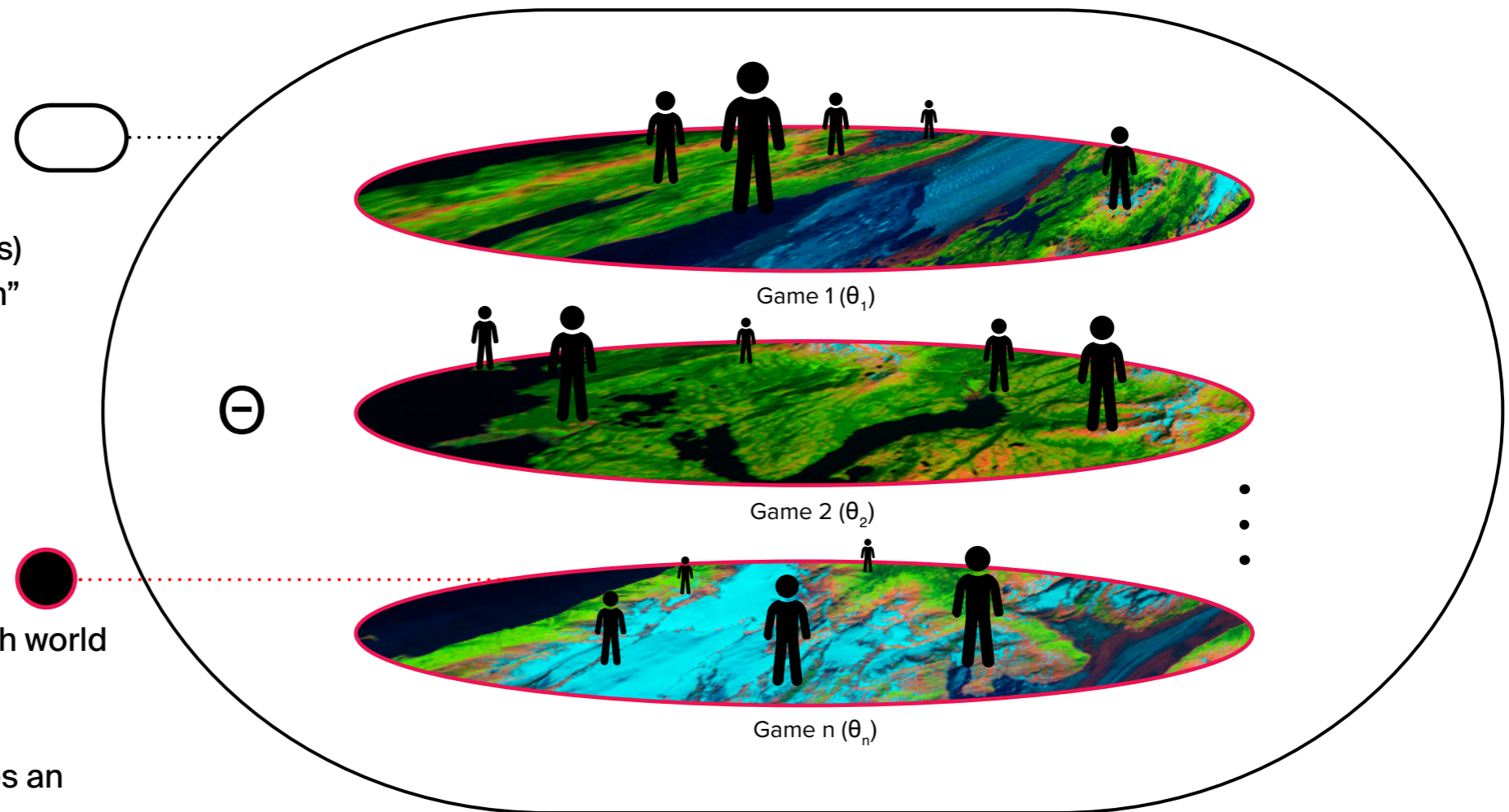
Empirical Mechanism Design - Schematic

Parameter search

- A set Θ describes all possible worlds (games)
- θ in Θ is a “mechanism”

Equilibria estimation

- Multiple players in each world
- Find an equilibria with chosen θ
- The designer measures an objective at equilibrium



Empirical Mechanism Design - Notation

Fix some parametrizable mechanism, (e.g., a first-price auction).

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Empirical Mechanism Design - Problem Statement

$f(\vec{s}; \Gamma_\theta)$ is the designer's objective function (e.g., revenue) evaluated at profile \vec{s} in game Γ_θ , where $f(\vec{s}; \Gamma_\theta) \in \mathbb{R}$

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The mechanism designer's problem is to find θ^* such that:

$$\theta^* \in \arg \max_{\theta \in \Theta} F(\theta; \Gamma_\theta)$$

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Solving the mechanism designer problem requires computing $E(\Gamma_\theta)$, the set of equilibria of a θ -simulation-based game, Γ_θ .

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But, computing Nash equilibria (even just one) is intractable, but sometimes feasible for small games (recall Gambit).

Consequently, we explore alternative solution concepts.

Challenge: find a solution concept that is approximable and tractable.

Strongly Connected Components Approximation Result

Theorem: (Recall-Precision)

■ If G_1 is an ϵ -**uniform approximation** of game G_2 , then

- Every SCC of G_1 is a 2ϵ -SCC of G_2
- Every 2ϵ -SCC of G_2 is a 4ϵ -SCC of G_1

(ϵ -SCC of a game allows for ϵ -edges of the better-response graph)

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Solution Concept	Approximable?	Tractable?	Existence?
Mixed Nash	✓	✗	Always
Pure Nash	✓	✓	Sometimes
Sink	✗	✓	Always
SCC	✓	✓	Always

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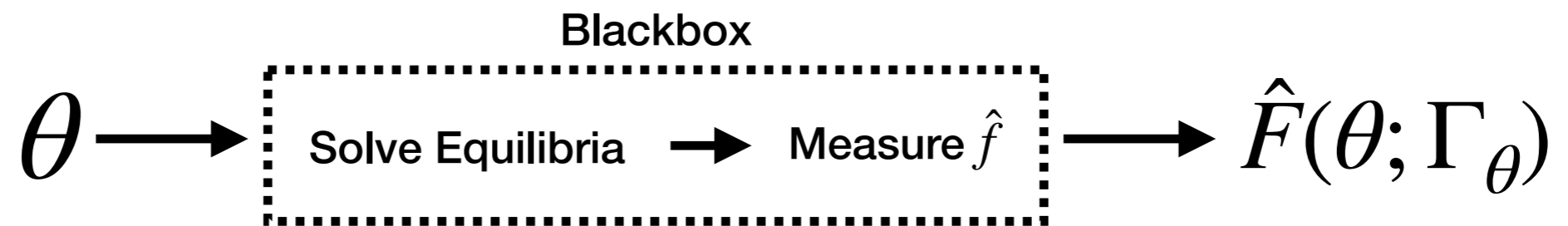
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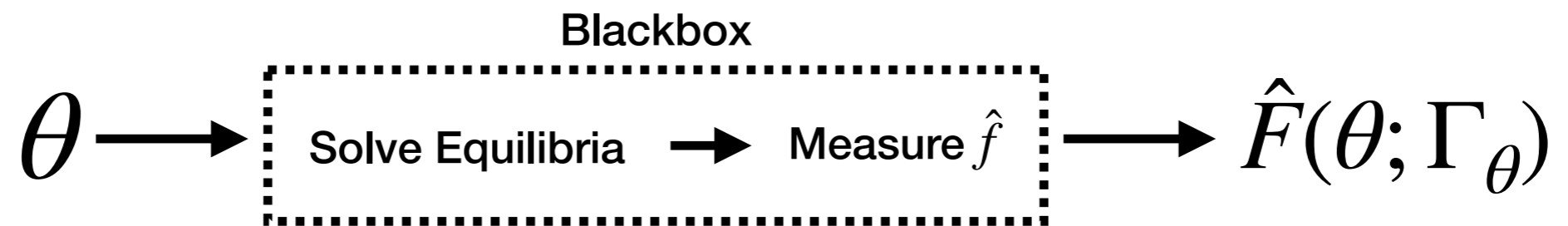
Black-Box Optimization

In case the design space is finite ($|\Theta| < \infty$), we derive an algorithm that provably learns approximately optimal mechanism's parameters.

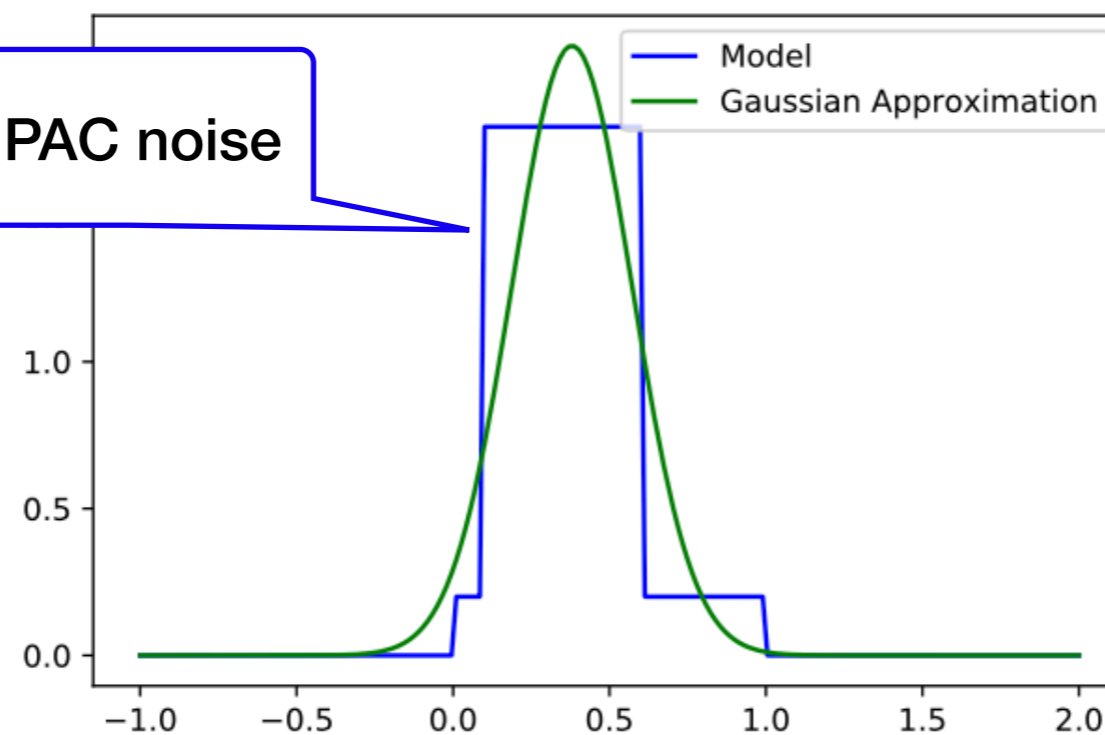
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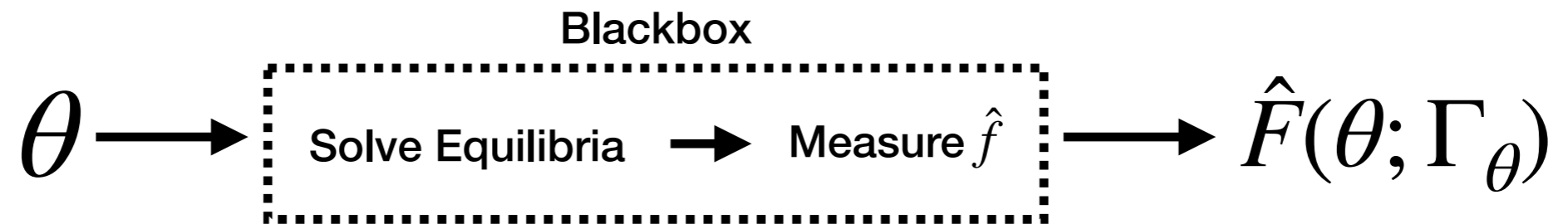
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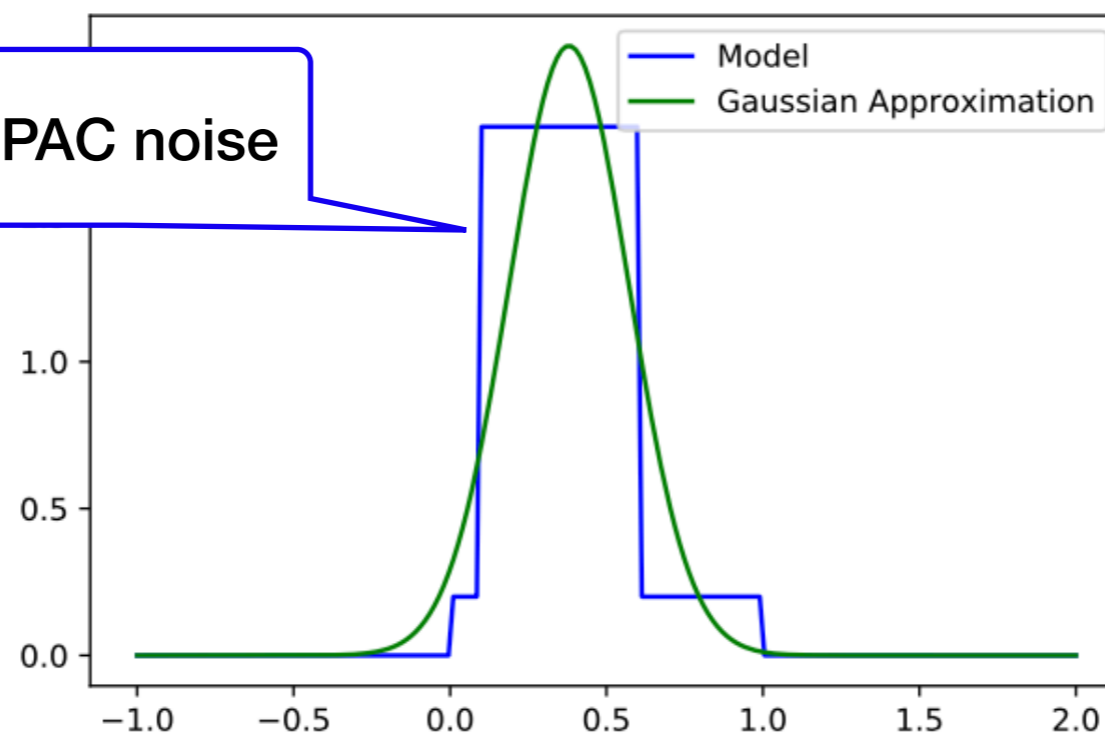
We observe $\hat{F}(\theta; \Gamma_\theta)$ with PAC noise



Black-Box Optimization

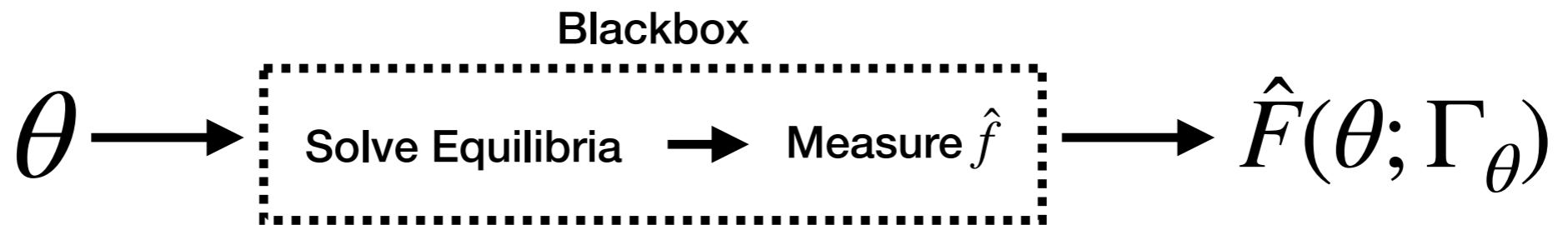


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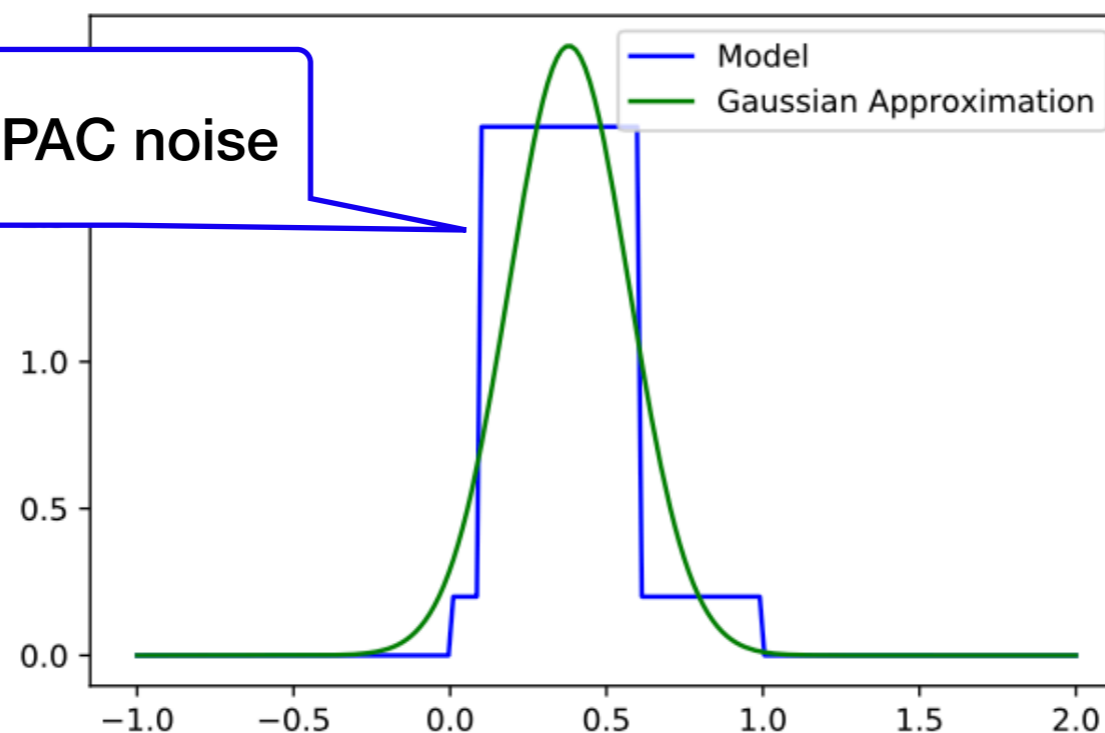


The best Gaussian approximation (minimum Kullback-Leibler divergence) of a 90% confidence interval on $[0.1, 0.6]$ where F ranges over $[0, 1]$.

Black-Box Optimization



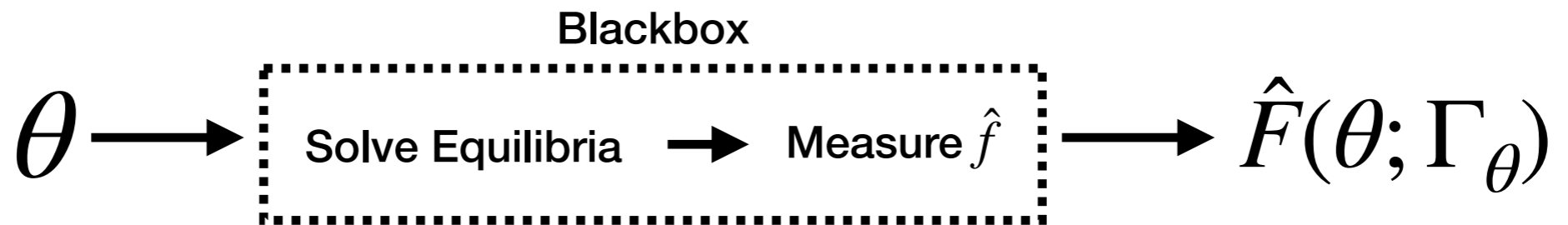
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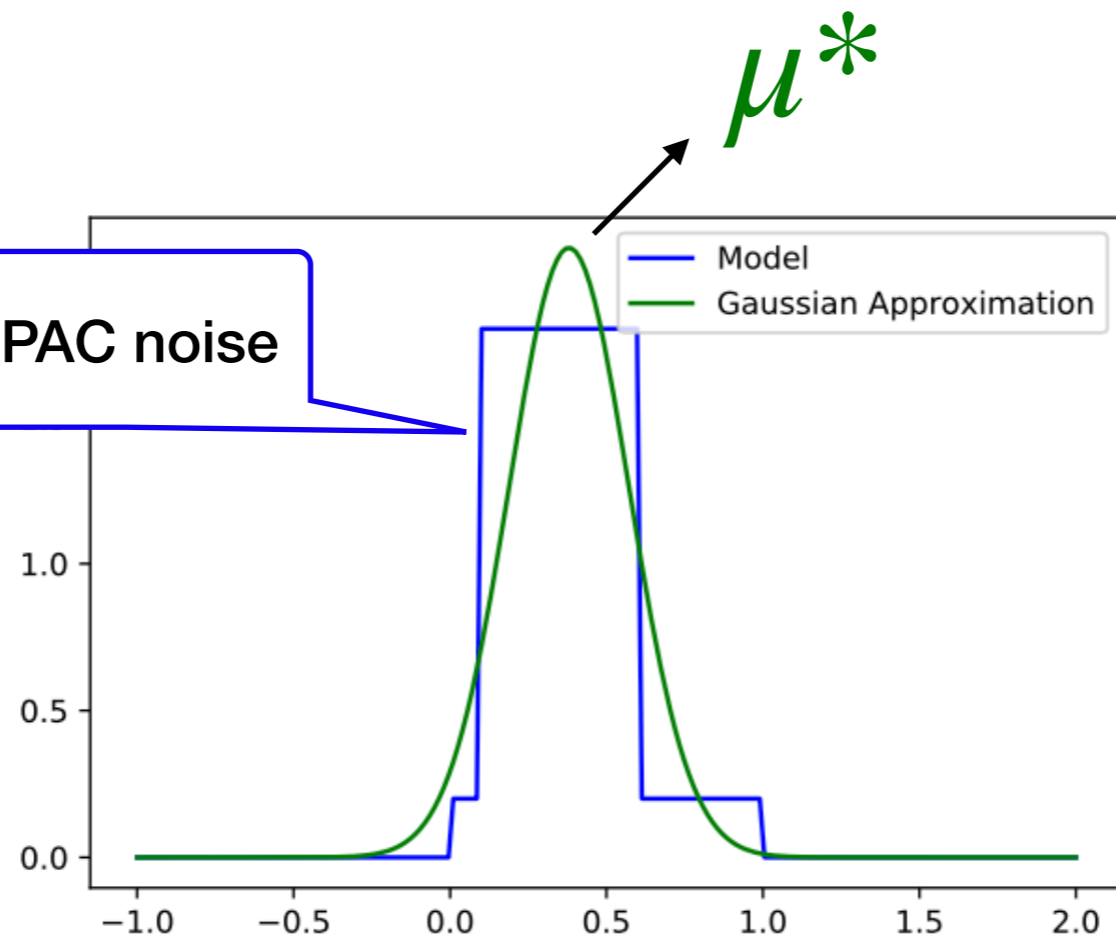
The best Gaussian approximation (minimum Kullback-Leibler divergence) of a 90% confidence interval on $[0.1, 0.6]$ where F ranges over $[0, 1]$.

Our Bayesian optimization search algorithms for EMD uses either the fitted gaussian (μ^*, σ^*) to PAC noise, or only the mean $(\mu^*, \sigma = 0)$, or the PAC noise mean directly $(\mu, \sigma = 0)$ as the measurement of the objective function, F .

Black-Box Optimization



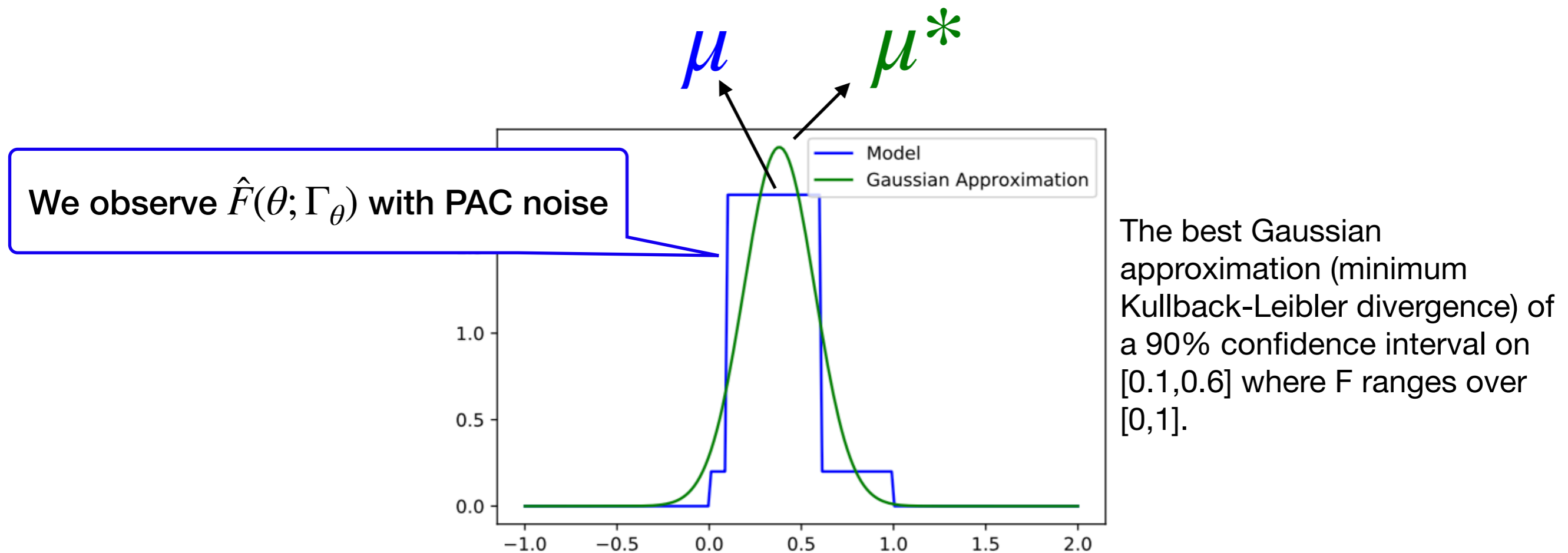
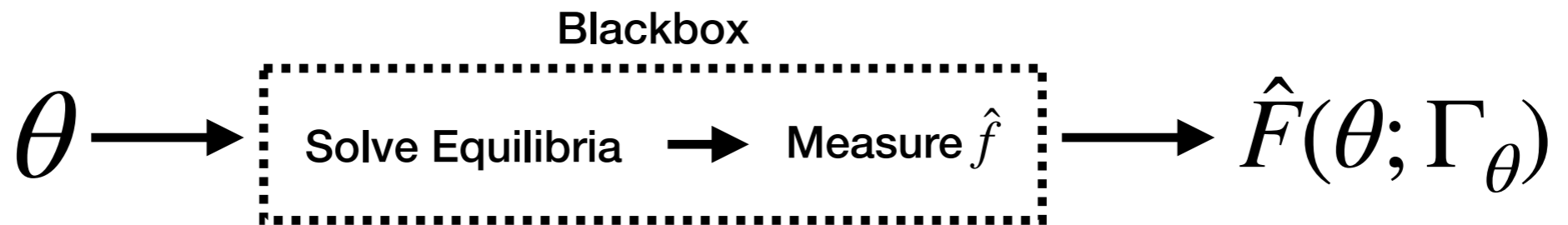
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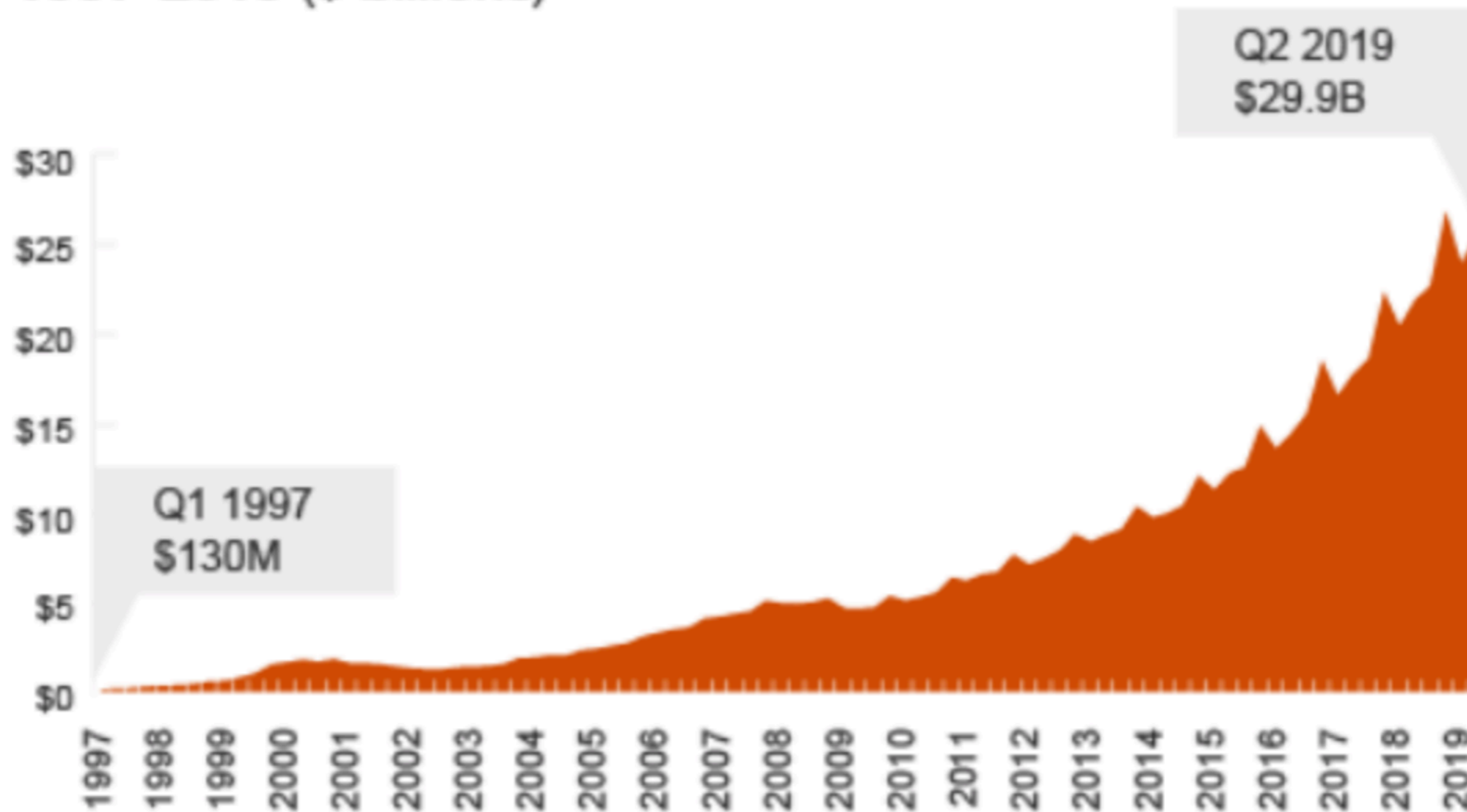
Electronic Advertisement Auctions

Electronic Advertisement Auctions



Electronic Advertisement Auctions

**Quarterly internet advertising revenue growth trends
1997-2019 (\$ billions)**



Source: IAB/PwC Internet Ad Revenue Report, HY 2019

Electronic Advertisement Exchanges

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- Advertisers might have different **objectives**, e.g., to immediately convert clicks into purchases, or to maintain brand awareness.

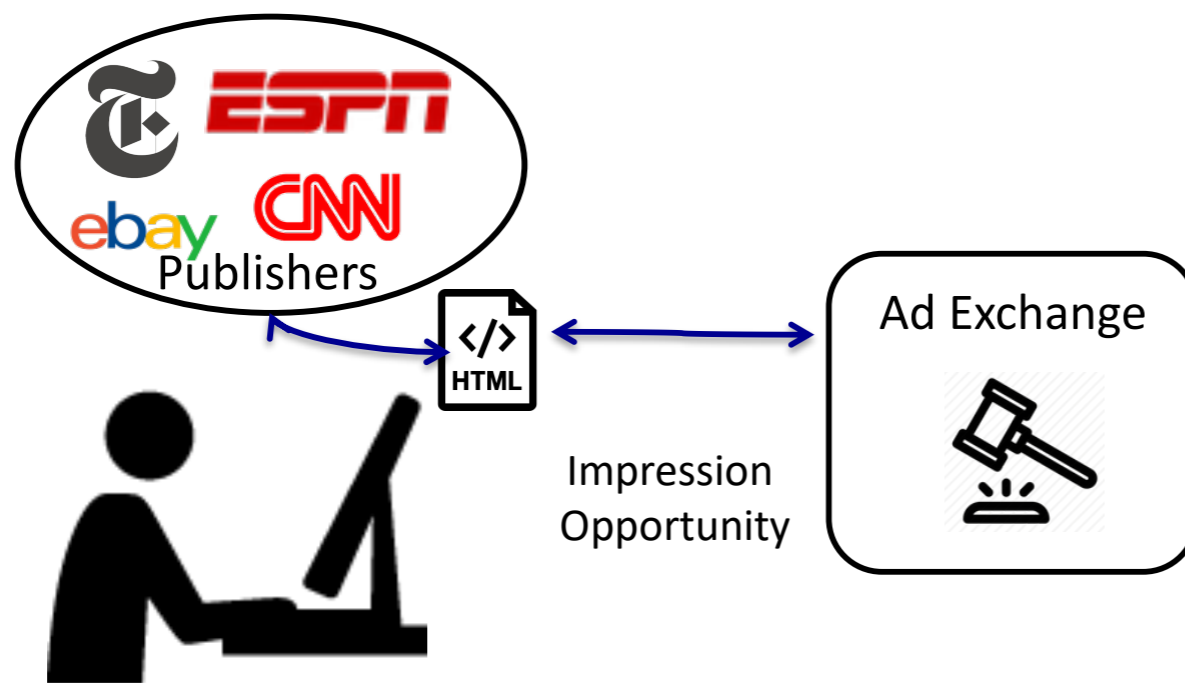
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- Advertisers might have different **objectives**, e.g., to immediately convert clicks into purchases, or to maintain brand awareness.
- We focus on **brand-awareness advertisement** where advertisers need to reach a certain number of potential customers, from certain demographics, for a fixed (pre-determined) budget

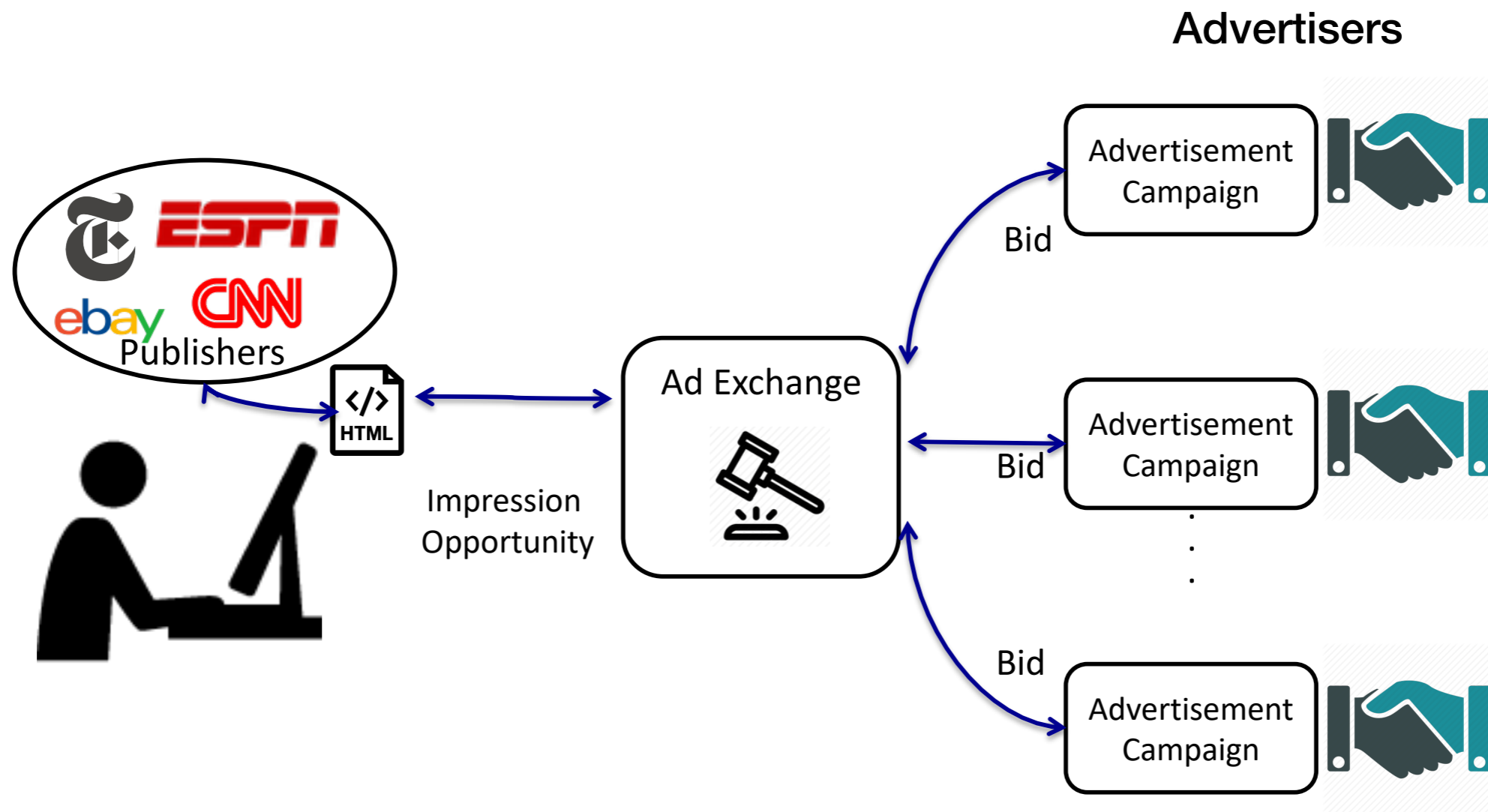
Electronic Advertisement Exchanges - Schematic



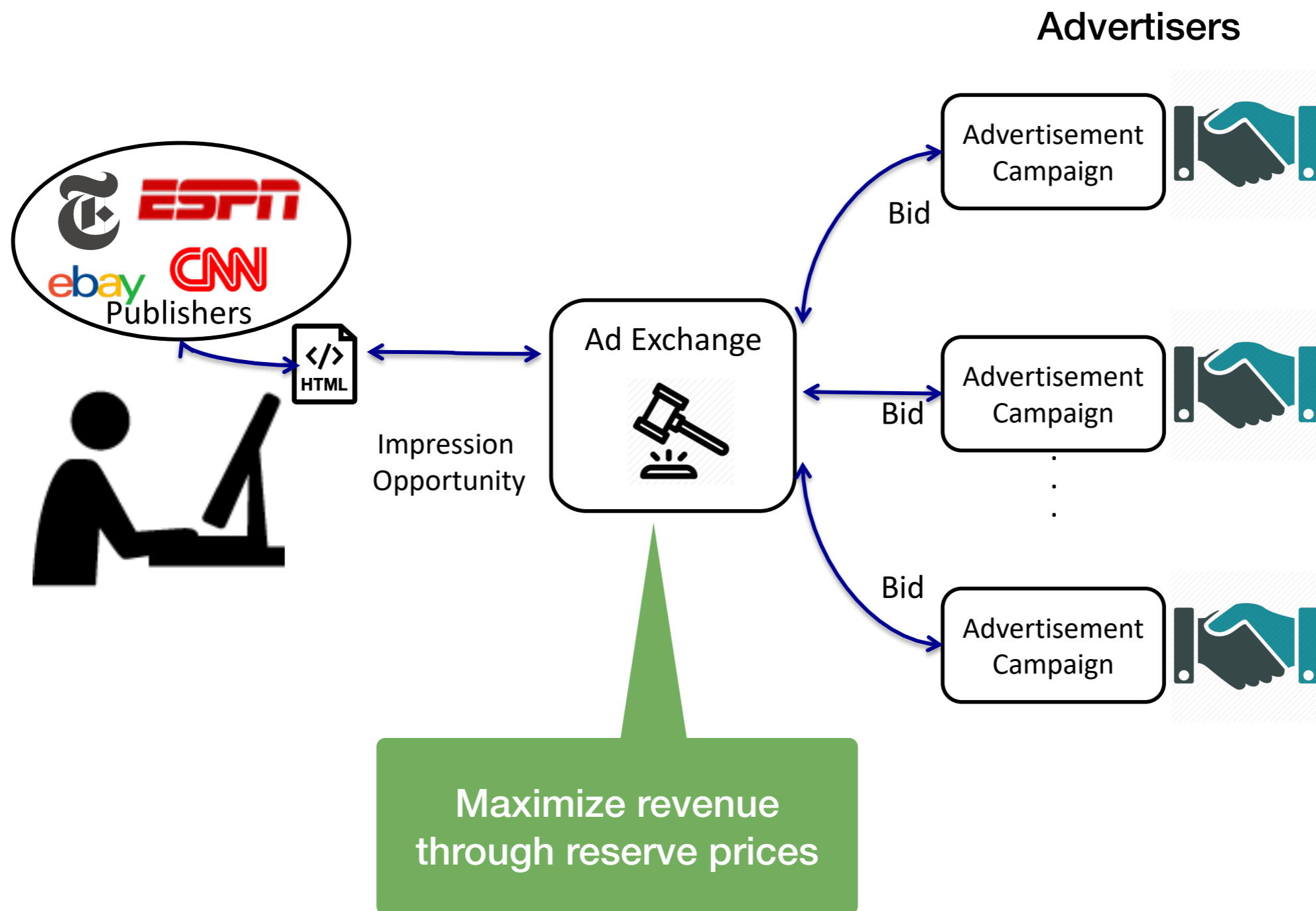
Electronic Advertisement Exchanges - Schematic



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Electronic Advertisement Exchanges - Schematic



Electronic Advertisement Exchanges - Model

- **Stage 1:** the *ad exchange* announces $\vec{r} \in \mathbb{R}_+^m$, where $\langle r_1, \dots, r_m \rangle \in \Theta$ is such that r_j is the reserve price for the j^{th} demographic or market segment.

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- **Stage 3:** some fixed number of *impression opportunities* arrive, where the demographic of each is drawn from some distribution.

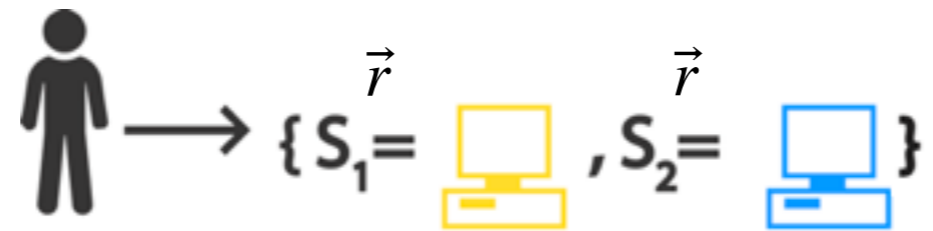
Electronic Advertisement Exchanges - Model

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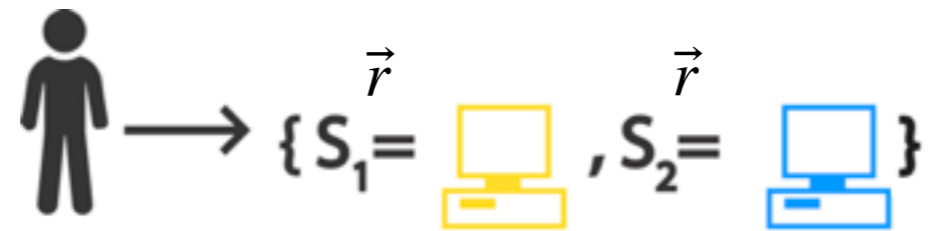
Electronic Advertisement Exchanges - Heuristics




We devised two heuristics for our experimental setup.



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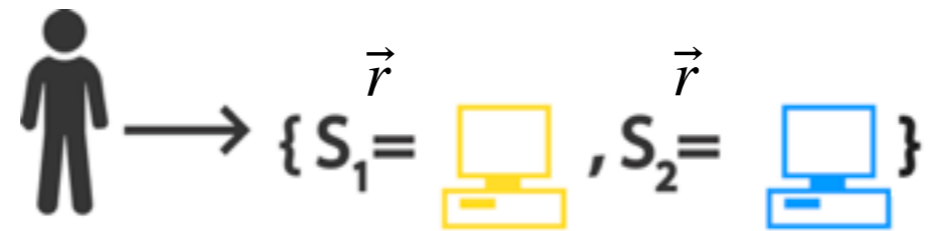


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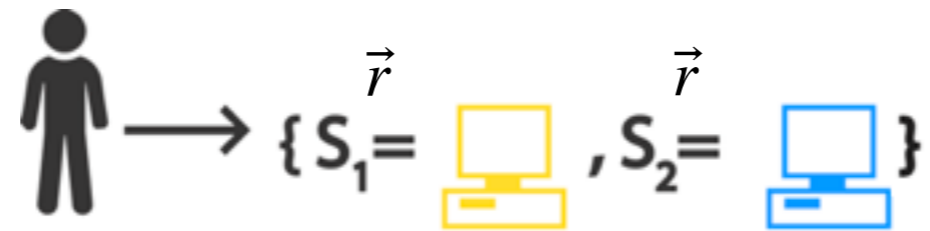


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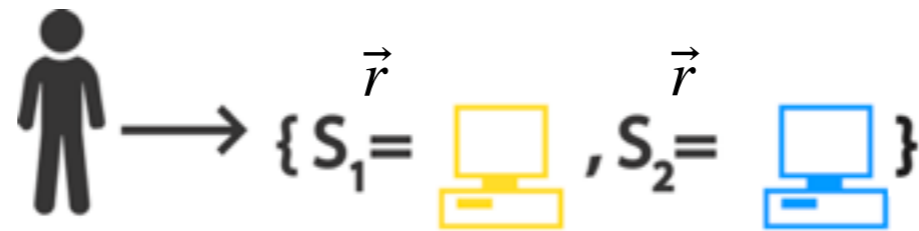
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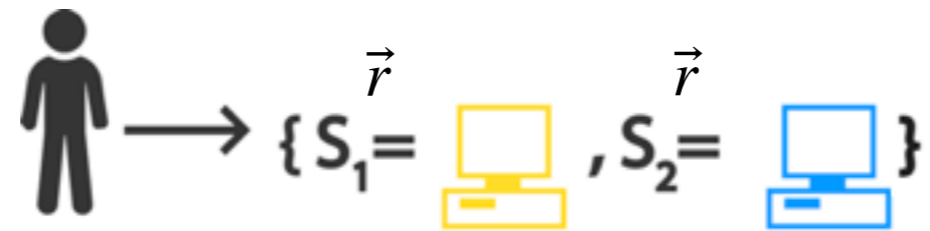
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- Bidding based on simulating the ad exchange dynamics.

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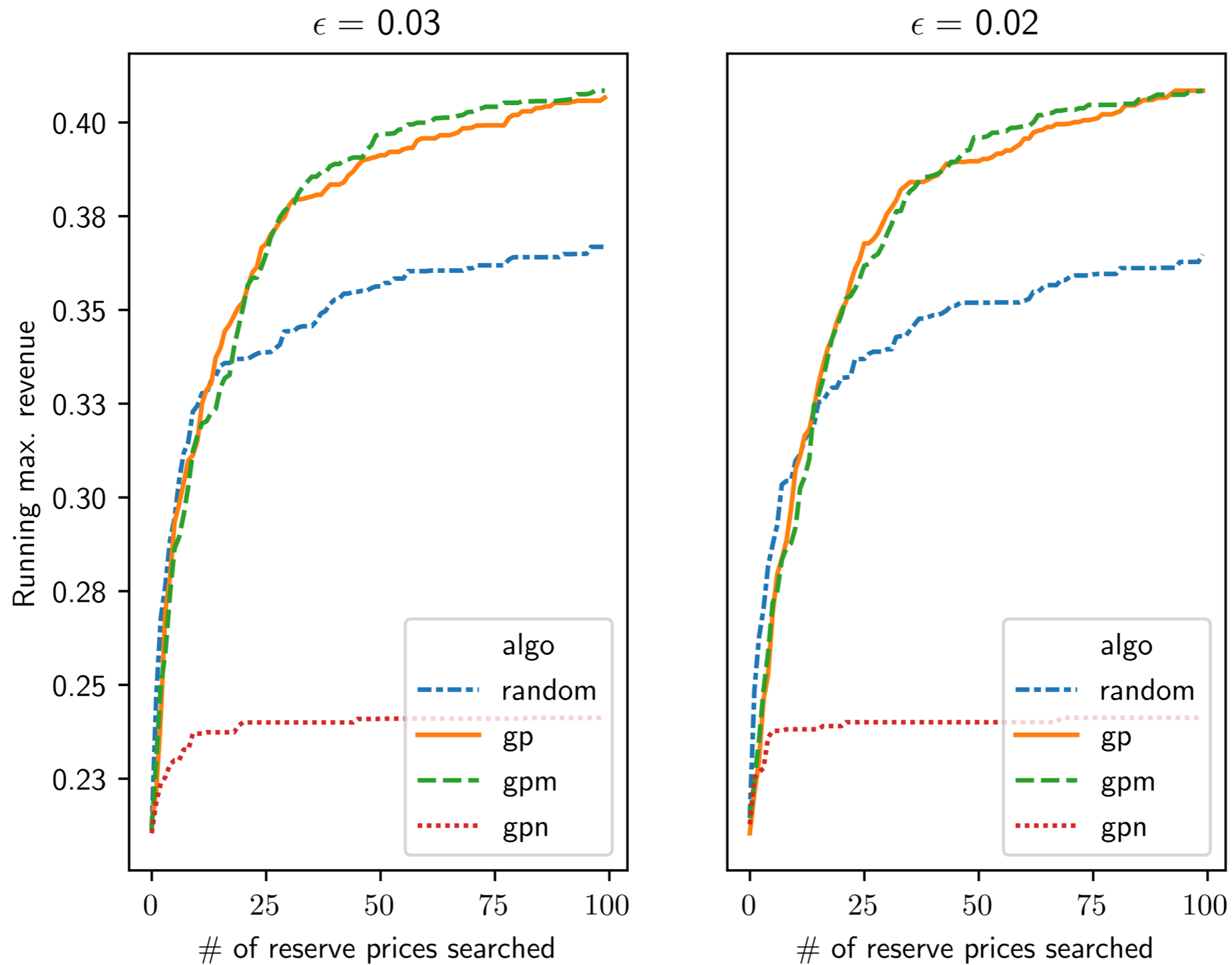
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- The task then if to find an 8-dimensional vector of reserve prices $\vec{r}^* \in \Theta$ that maximizes the ad exchange revenue, at equilibrium.

Experimental Results

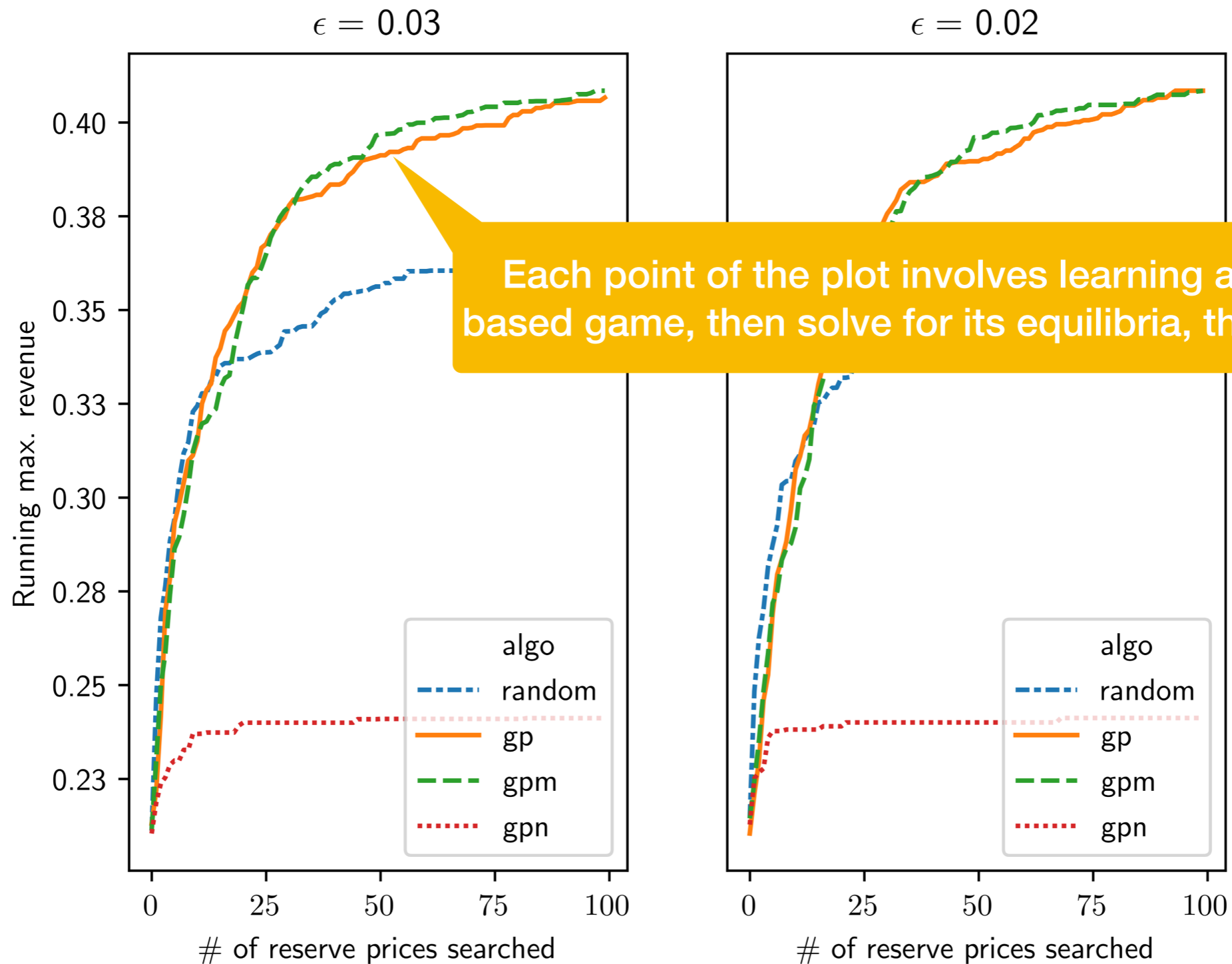
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- We empirically showed the effectiveness of our BO algorithms in a styled but rich simulation of electronic advertisement exchanges.

Part 3:

Proposed Work

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- **Propose:** extend our simulation-based game methodology to computing competitive equilibria in **noisy combinatorial market**

Preliminary Work:

Learning Competitive Equilibria in Noisy Combinatorial Markets.
Enrique Areyan Viqueira and Amy Greenwald.
2nd Games, Agents, and Incentives Workshop (GAIW@AAMAS 2020)

Autonomous Negotiation Agents

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- **Currently:** mentoring a group of undergraduate students to participate in 2020's ANAC SCML competition.

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Timeline

<i>Task</i>	<i>Date</i>
Noisy Combinatorial Markets	Summer/Fall 2020
Autonomous Negotiation Agents	Summer 2020
Thesis Writing	Spring 2021
Thesis Defense	May 2021

Thank you for your attention!

Thesis Statement

Through modern statistical tools, sampling heuristics, and optimization techniques, we find sample-efficient algorithms that learn the approximate equilibria of simulation-based games and use them to empirically design mechanisms.