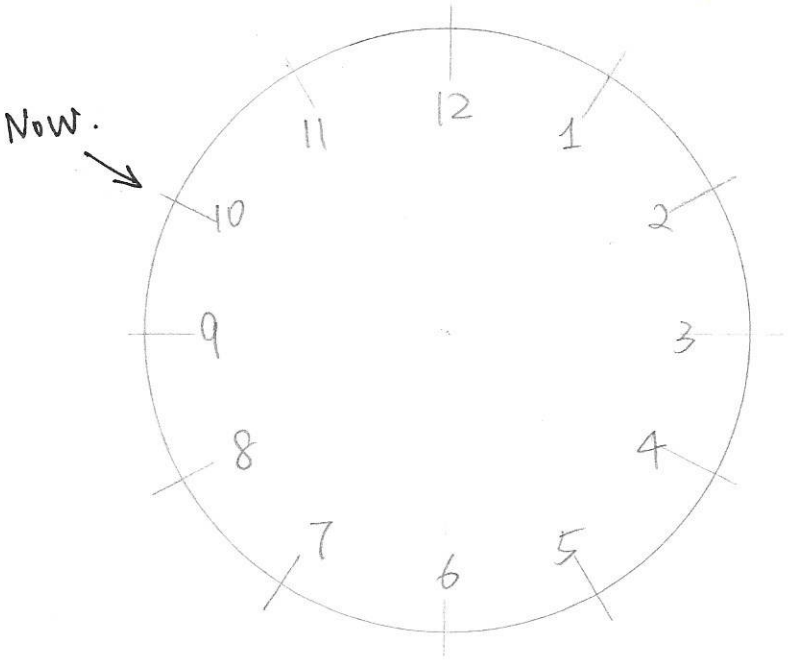
Congruence



Now. 10 am.

Lets meet 3 hours later. when will we meet? $10+3=\boxed{13}$ (a.m.) ?

↳ 1 (p.m.)

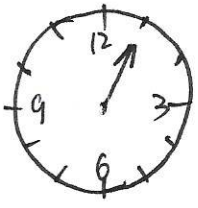
Lets meet 15 hours later. $10+15=\boxed{25}$

10 am $\xrightarrow[12\text{ hrs}]{}$ 10 pm $\xrightarrow[2\text{ hrs}]{}$ 12 am $\xrightarrow[1\text{ hrs.}]{}$ 1 am

Lets meet 27 hours later $10+27=\boxed{37}$

10 am $\xrightarrow[24\text{hrs}]{}$ 10 am $\xrightarrow[3\text{ hrs}]{}$ 1 pm.

1 a.m. 1 p.m. are like this in clock.



$13 = 12 \times 1 + \boxed{1}$

$25 = 12 \times 2 + \boxed{1}$

$37 = 12 \times 3 + \boxed{1}$

↑ same remainder.

$13 \equiv 25 \pmod{12}$

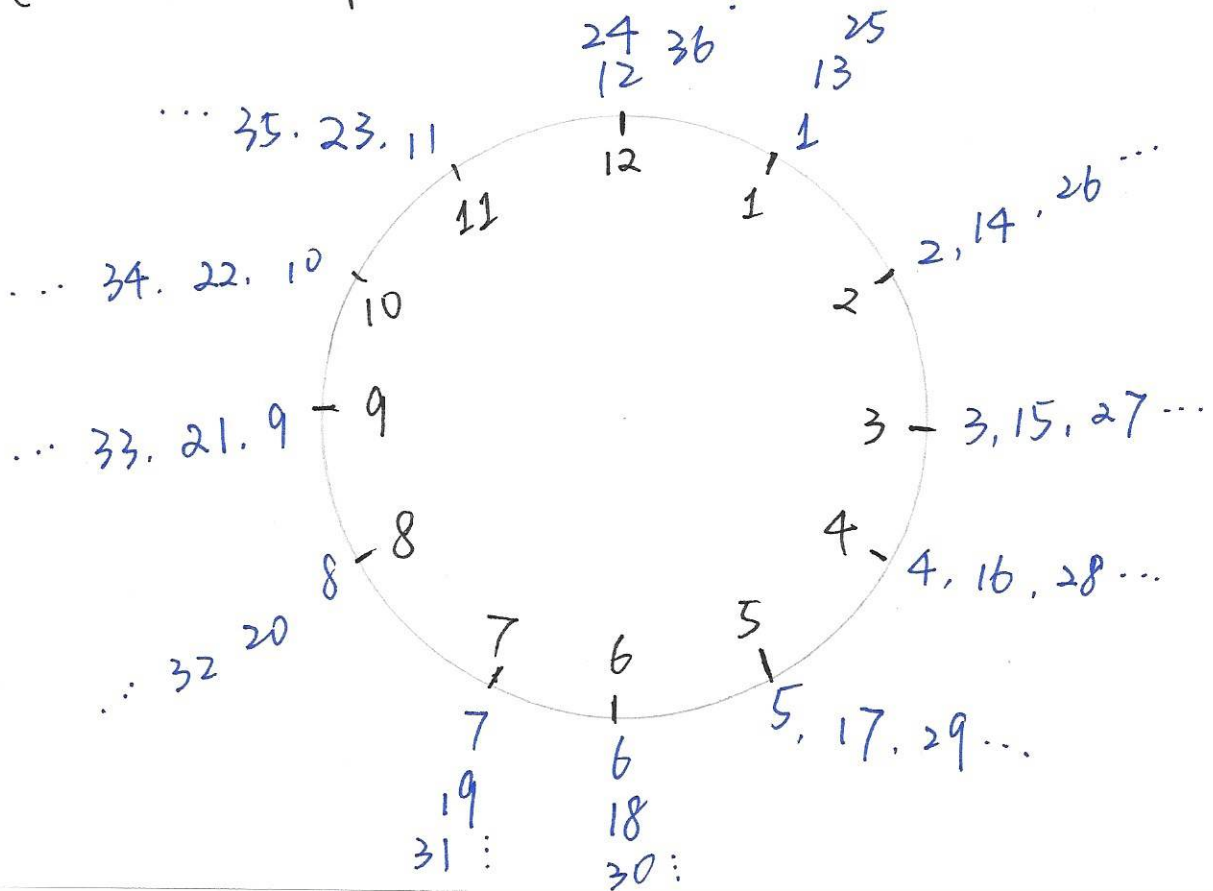
$25 \equiv 37 \pmod{12}$

$13 \equiv 1 \pmod{12}$

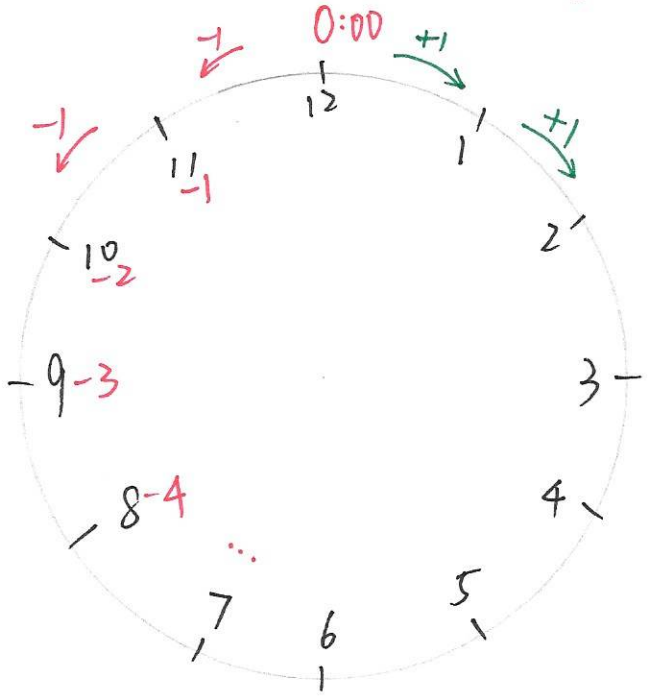
$25 \equiv 1 \pmod{12}$

$37 \equiv 1 \pmod{12}$

Classify ~~numbers~~ positive integers in congruence class (have same expression in clock).



Question: where do we put negative integers ?



Congruence class

⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13
-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
representatives → 0	1	2	3	4	5	6	7	8	9	10	11
12	13	14	15	16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31	32	33	34	35
36	37	38	39	40	41	42	43	44	45	46	47
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

-12, -12, +12, +12, +12

12 classes all together (mod 12)

Generally. n classes (mod n)

Practice :

(1) $5 \not\equiv 100 \pmod{12}$

No! $5 \xrightarrow{+12} 17 \xrightarrow{+12} 29 \xrightarrow{+12} 41 \xrightarrow{+12} 53$

$\xrightarrow{+12} 65 \xrightarrow{+12} 77 \xrightarrow{+12} 89 \xrightarrow{+12} 103$

(pass 100)

(2) $5 \not\equiv -7 \pmod{12}$

Yes! $-7 \xleftarrow{-12} 5$

(3) $2 \not\equiv 14 \pmod{12}$

Yes!

(4) $26 \not\equiv 14 \pmod{12}$

Yes!

(5) $1 \not\equiv -1 \pmod{12}$

No!

Faster method

$$a \not\equiv b \pmod{12}$$

Check whether $a-b$ is multiple of 12

$\begin{cases} \text{Yes} \longrightarrow \text{congruent} \\ \text{No} \longrightarrow \text{not congruent.} \end{cases}$