Integers
Activity 8
Math-T101 Spring 2014

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1 Negative numbers

What are negative numbers? How are they defined?

The numbers less than zero are called negative numbers.

The set of integers is: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9

negative numbers — positive numbers

integers

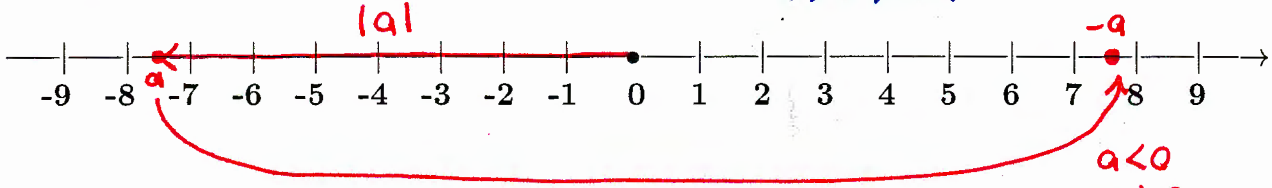
Applications of integers:

* Thermometer ⇒ $-10°F$, $20°C$
* Elevation ⇒ above / below sea level
* Bank accounts ⇒ credits / debits

1.1 Order of integers

Problem 1. *Place a, b, and c on the number line given $a \leq 0 \leq b \leq c$. Then place $-a$, $-b$, and $-c$ on the number line.*

$-a \geqslant 0 \geqslant -b \geqslant -c$



1.2 Absolute value

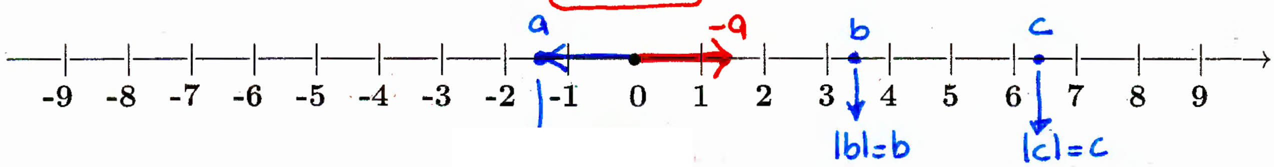
Definition of absolute value: Distance between the number and zero in the numberline

$|b| = b$ if $b > 0$
$|b| = -b$ if $b < 0$

$\Big\}$ $|b|$ is the distance from b to 0.

Problem 2. *Place a, b, and c on the number line given $a \leq 0 \leq b \leq c$. Then place $|a|$, $|b|$, and $|c|$ on the number line.*

$\boxed{|a| = -a}$



Problem 3. *Determine if the statement is true for all pairs of integers (a, b), or true for some and false for others, or false for all pairs of integers.*

a) $|a + b| = |a| + |b|$ → True for some, false for some others

$a=3$, $b=5$: $|a+b| = |3+5| = |8| = 8$
$|3| + |5| = 3 + 5 = 8$ =

$a=3$, $b=-5$: $|a+b| = |3 + (-5)| = |(-2)| = 2$
$|3| + |-5| = 3 + 5 = 8$ ≠

b) $|a + b| \leq |a| + |b|$ → True for all. This is called triangle inequality.

c) $|a - b| = |b - a|$ → True for all

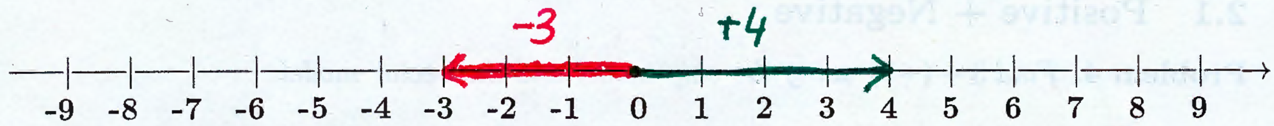
$|a-b| = a-b$ if $a-b > 0$
then $a > b$
then $0 > b-a$

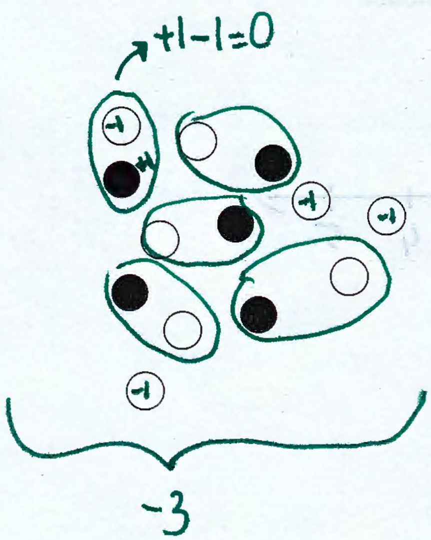
$|b-a| = -(b-a)$
$= a-b$

=

$|a-b| = -(a-b) = b-a = |b-a|$
if $a-b < 0$
$a < b$
then $0 < b-a$

1.3 Models for integer representation

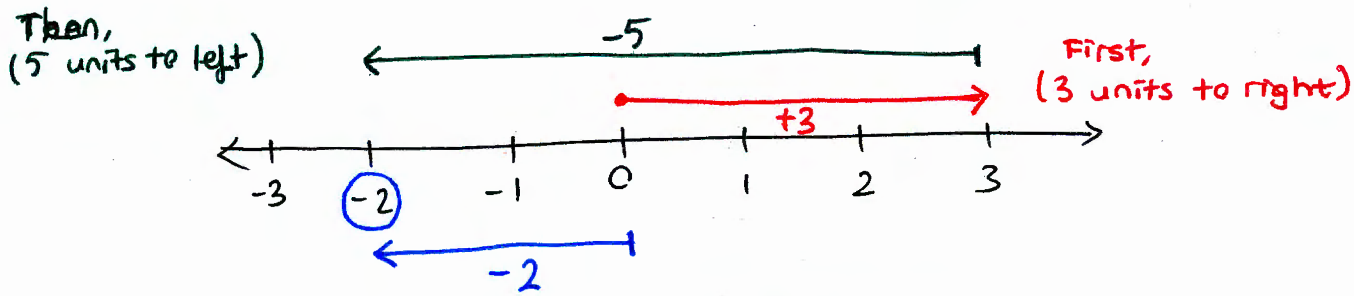




2 Addition

2.1 Positive + Negative

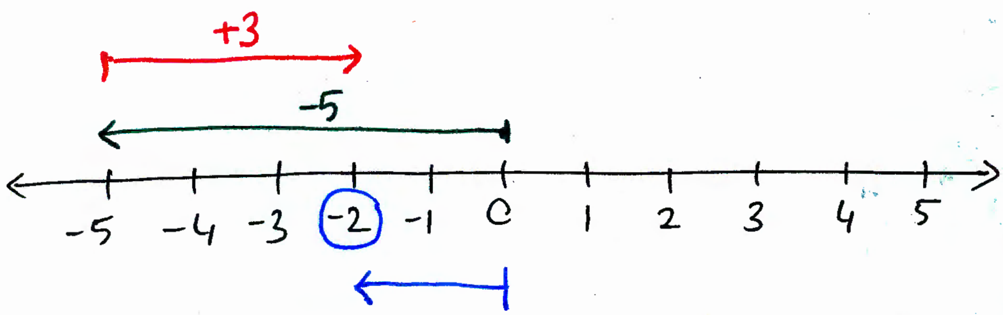
Problem 4. *Find* $3 + (-5)$ *using the chip model and the vector model.*



$$3 + (-5) = -2$$

2.2 Negative + Positive

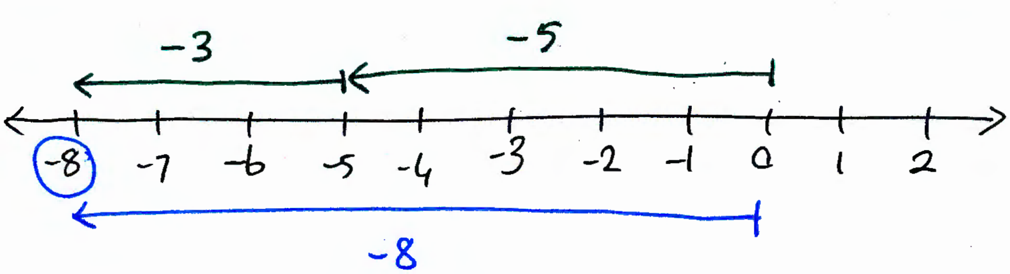
Problem 5. *Find* $-5 + 3$ *using the chip model and the vector model.*



$$-5 + 3 = -2$$

2.3 Negative + Negative

Problem 6. *Find* $-5 + (-3)$ *using the chip model and the vector model.*



$$-5 + (-3) = -8$$

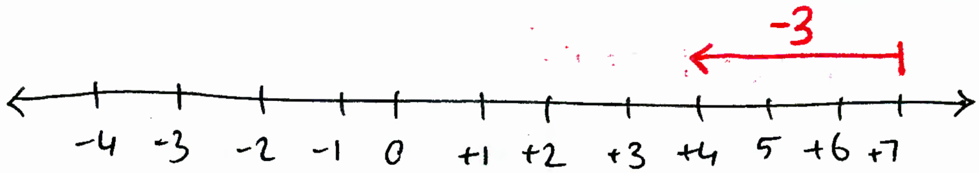
3 Subtraction

Pay attention to the definition of subtraction as finding the missing addend. It helps with understanding integer subtraction.

3.1 Positive - Positive

Problem 7. *Find* $4-7$ *using the chip model and the vector model.*

$7 + \boxed{} = 4$ What number we add 7 to get 4?

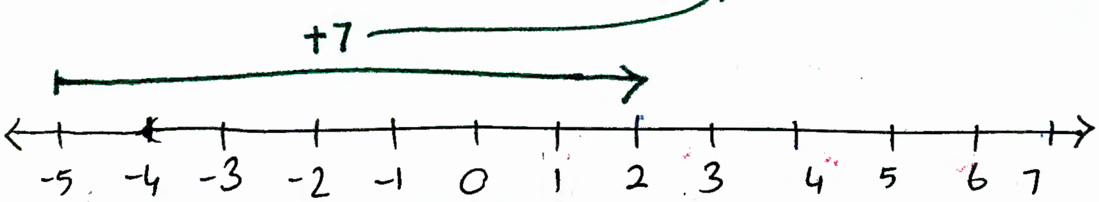


$4-7=-3$

3.2 Positive - Negative

Problem 8. *Find* $2-(-5)$ *using the chip model and the vector model.*

$(-5) + \boxed{} = 2$ What number do we add -5 to get 2?

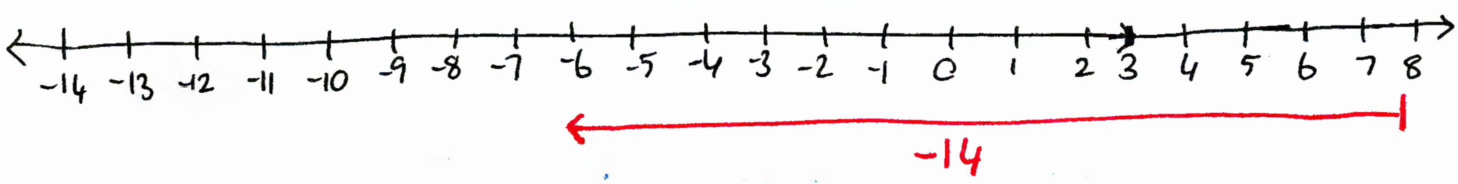


$2-(-5)=7$

3.3 Negative - Positive

Problem 9. *Find* $-6-8$ *using the chip model and the vector model.*

$8 + \boxed{} = -6$ what number do we add 8 to get -6?

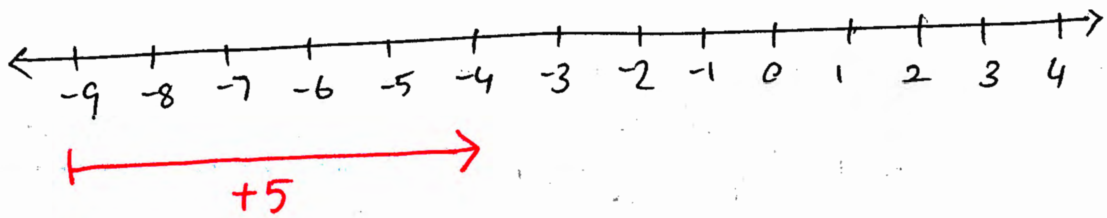


$-6-8=-14$

3.4 Negative - Negative

Problem 10. *Find* $-4-(-9)$ *using the chip model and the vector model.*

$-9+\boxed{}=-4$ What number do we add -9 to get -4?



$-4-(-9)=+5$

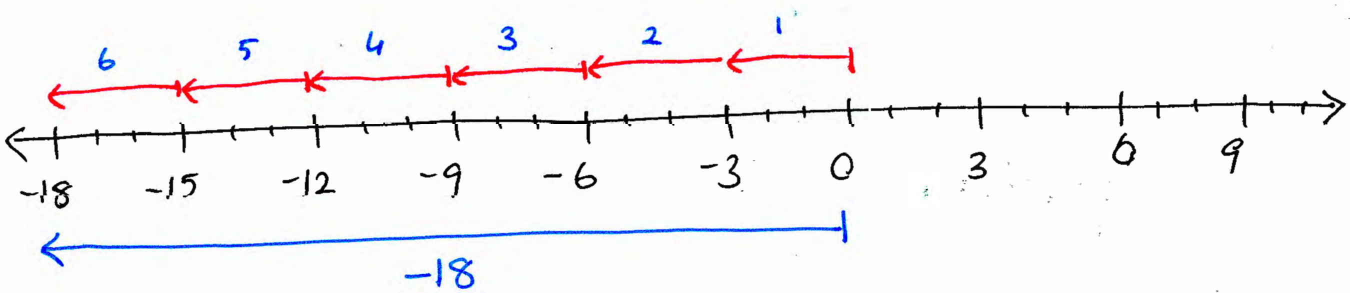
4 Multiplication

Unfortunately, models for multiplication and division involving negative numbers are not as intuitive as models for positive numbers. One can think about it either in terms of time or temperature to make sense, but even then it is a bit more challenging. The clearest way of understanding seems to be observing the pattern. The chip model is not a good model to use for either multiplication or division involving negative numbers.

4.1 Positive × Negative

The repeated addition definition still works here.

Problem 11. *Find* $6\times(-3)$ *using repeated addition. Show your work on the number line.*



$6\times(-3)=-18$

4.2 Negative × Positive

Because of the commutative property of multiplication this is exactly as Positive × Negative.

$(-3)\times 6=6\times(-3)$ check above

4.3 Negative × Negative

positive x negative

positive x positive

×	-4	-3	-2	-1	0	1	2	3	4
4	-16	-12	-8	-4	0	4	8	12	16
3	-12	-9	-6	-3	0	3	6	9	12
2	-8	-6	-4	-2	0	2	4	6	8
1	-4	-3	-2	-1	0	1	2	3	4
0	0	0	0	0		0	0	0	0
-1	4	3	2	1	0	-1	-2	-3	-4
-2	8	6	4	2	0	-2	-4	-6	-8
-3	12	9	6	3	0	-3	-6	-9	-12
-4	16	12	8	4	0	-4	-8	-12	-16

negative x negative

negative x positive

5 Division

It is best to think about division as finding the missing factor. In that case, we see that the rules for multiplication apply to division as well.

5.1 Positive ÷ Negative

Problem 12. *Find* $32 \div (-8)$ *and explain in terms of finding the missing factor.*

$(-8) \times \boxed{-4} = 32$

((-8) times what makes 32? - check multiplication table)

Q.1 ⇒ what is the sign of missing factor?

Multiplication table tells that it is negative (-)

Q.2 ⇒ (Ignore signs) How many 8 makes 32? (4)

5.2 Negative ÷ Positive

Problem 13. *Find* $-42 \div (6)$ *and explain in terms of finding the missing factor.*

$6 \times \boxed{} = -42$

Q.1 ⇒ what is the sign of missing factor?

multip. Table tells that it is negative (-)

Q.2 ⇒ How many 6 makes 42 (by ignoring signs)?

7

-7

5.3 Negative ÷ Negative

Problem 14. *Find* $-54 \div (-9)$ *and explain in terms of finding the missing factor.*

$\rightarrow (-9) \times \boxed{} = -54$

Q.1 ⇒ What is the sign of the missing factor?

Positive

Q.2 ⇒ How many 9 makes 54?

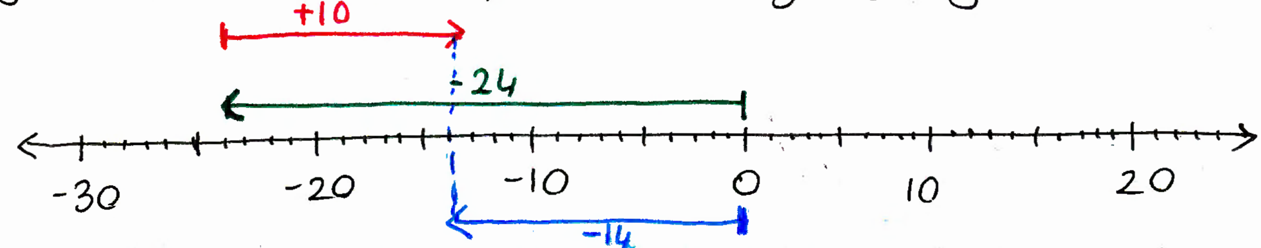
6

+6

6 Problems

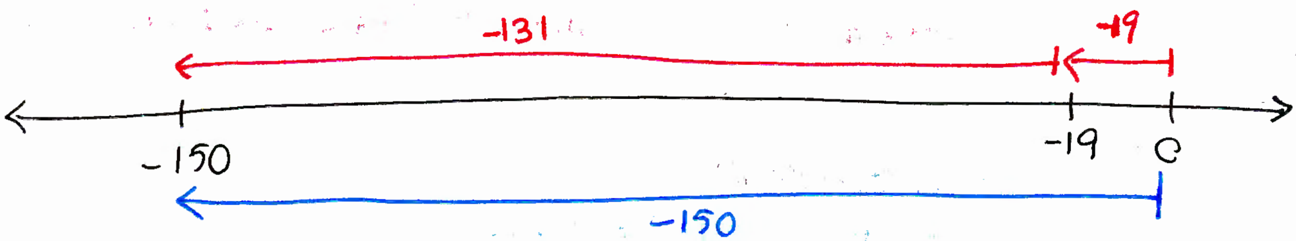
Problem 15. *Create a word problem for -24 + 10, using the vector model.*

The temperature at midnight was -24°C. It increased 10°C during the day. What was the temperature during the day?



Problem 16. *Create a word problem for -19 + (-131), using the money model.*

Carla's credit card statement showed that she owed $19. She charged $131 for gasoline. What is her new balance?



Problem 17. *Find* $|0|$, $|25-7|$, *and* $|7-25|$.

$|0|=10$

$|25-7| = |18| = 18$

$|7-25| = |-18| = 18$

The absolute value of a number is always positive with the exception of the absolute value of 0 because |0|=0 and 0 is neither positive nor negative.

Problem 18. *For which numbers a is $-a$ positive? For which is $-|a|$ positive?*

$\boxed{\dfrac{a<0}{-a>0}}$ $\qquad \dfrac{a>0}{-a<0} \qquad \dfrac{a=0}{-a=0}$

$|a|>0 \qquad |a|>0 \qquad |a|=0$

$\boxed{-|a|<0 \qquad -|a|<0}$ → $-|a|$ can not be positive

Problem 19. *A student claims that $|a+b| \le a+b$ for all integers a and b. Explain why the student is wrong.*

Counter example:

$a=-5,\quad b=2 \implies |a+b| = |-5+2| = |-3| = 3$

$a+b = (-5)+(2) = (-3)$

$3 > -3$

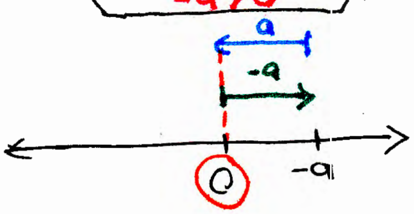
$|a+b| > a+b$

For $a+b>0$ then $|a+b| = a+b$

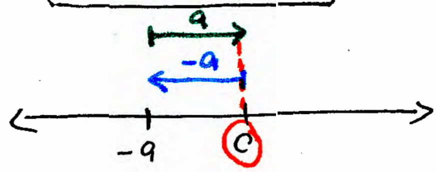
If $a+b<0$, then check $a>0,\ b<0,\ b>a$ & $a<0,\ b>0,\ a>b$

Problem 20. *Use the vector model to verify the following identities: $(-a)+a=0$ (draw one picture for $a>0$, another for $a<0$). What happens if $a=0$?*

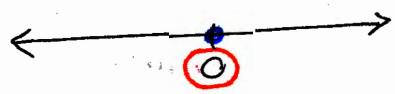
$\dfrac{a<0}{-a>0}$



$\dfrac{a>0}{-a<0}$



$\dfrac{a=0}{-a=0}$

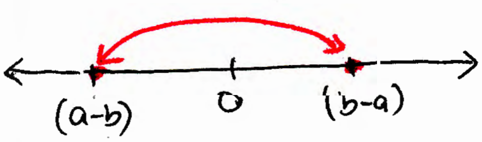


$-(a-b) = b-a$ *(draw one picture for $a<b$ and another for $b<a$).*

$\dfrac{a<b}{a-b<0}$

$-(a-b)>0$

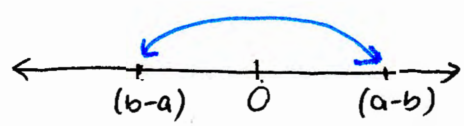
$b-a>0$



$\dfrac{b<a}{0<a-b}$

$0>-(a-b)$

$0>b-a$



Problem 21. *Simplify the following expressions:*

1. $(-46+34)\cdot 25-105$

$= -12 \cdot 25 - 105$

$= -300 - 105$

$= -405$

2. $-7\cdot(14-4)\div(17+(-12))$

$= -7 \cdot 10 \div 5$

$= -70 \div 5 = -14$

3. $x^2-(2x^2+(-3x))\cdot x+5x^2$

$= x^2 - 2x^3 + 3x^2 + 5x^2$

$= 9x^2 - 2x^3$

Problem 22. *Suppose a is a positive number and b is a negative number (so neither is zero). State whether the following are positive or negative.*

1. $(ab)^n$ *if n is odd* $\Rightarrow$ $\left.\begin{matrix} a>0 \\ b<0 \end{matrix}\right\}$ $a.b<0$

negative

$ab \cdot ab > 0$

$\underbrace{(ab.ab)\ldots\ldots(ab.ab)}_{n-1 \text{ times} = \text{even times}} > 0$

$\underbrace{ab.ab \ldots\ldots ab.ab.ab}_{n \text{ times} = \text{odd times}} < 0$

2. $(ab)^n$ *if n is even*

positive

3. $-ab^2$

$a>0$ $\quad b<0$

$-a<0$ $\quad b^2>0$

$-a.b^2$ is negative x positive, so it is negative

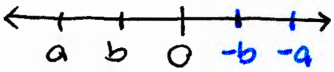
4. $(b-a)\cdot b$ $\rightsquigarrow$ if $a>0$ then $b-a = b+(-a) < 0$ $\Rightarrow$ $(b-a)\,b$ is negative times negative
$b<0$
so it is positive.

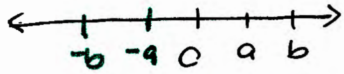
Problem 23. *Calculate* $3\frac{1}{8}-1\frac{5}{16}$ *using mental math strategies. Then use that to find* $1\frac{5}{16}-3\frac{1}{8}$.

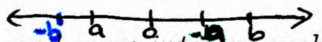
$1\frac{5}{16} - 3\frac{1}{8} = -\left(3\frac{1}{8} - 1\frac{5}{16}\right) = -\left(3\frac{2}{16} - 1\frac{5}{16}\right) = -\left(2\frac{18}{16} - 1\frac{5}{16}\right) = -\left(1\frac{13}{16}\right)$

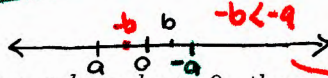
$= -1\frac{13}{16}$

Problem 24. *If a and b are integers such that $a < b$, then in what order do you think $-a$ and $-b$ would be? (That is, is $-a < -b$ or is $-b < -a$?) What about the order of $|a|$ and $|b|$?*

if $\left.\begin{array}{l} a<0 \\ b<0 \\ a<b \end{array}\right\}$ then  $-b<-a$ & $\left.\begin{array}{ll} -a>0 & |a|=-a \\ -b>0 & |b|=-b \end{array}\right\}$ $|b|<|a|$

if $\left.\begin{array}{l} a>0 \\ b>0 \\ a<b \end{array}\right\}$ then  $-b<-a$ & $\left.\begin{array}{l} |a|=a \\ |b|=b \end{array}\right\}$ $|a|<|b|$

if $a<0, b>0$ then $-b<-a$ 

OR $-b<-a$ 

$\left.\begin{array}{l} |a|=-a \\ |b|=b \end{array}\right\}$ $|a|<|b|$

$|a|>|b|$

Fact 1. *If a, b, and c are integers such that $a < b$ and $c > 0$, then $ac < bc$.*

Proof: Note that $a < b \Leftrightarrow b - a > 0$. Since $c > 0$, $(b - a)c$ is in the form positive × positive = positive. By applying the distributive property, we see that

multiplying both sides by a positive number does not change the direction of the inequality.

$(b-a).c > 0$

$bc - ac > 0$ (distributive property)

$bc - ac + ac > 0 + ac$ (adding both sides the same number does not change the direction of inequality)

$bc + 0 > 0 + ac$

$bc > ac.$ (additive identity)

→ multiplying both side by a negative number, it changes the direction of the inequality

Fact 2. *If a, b, and c are integers such that $a < b$ and $c < 0$ then $ac > bc$.*

Proof: If $a<b$, then $a-b<0$ & $c<0$

then $(a-b)\cdot c$ is in the form of negative × negative

= positive

$(a-b)\cdot c>0$

$ac-bc>0$ by distributive property

$ac>bc$.

Example 1. *Suppose we multiply the inequality $-1 < 4$ by 2? Do we reverse the direction of the inequality? What if we multiply $-4 < -1$ by -3?*

$-1<4$

$(-1)\cdot 2 < 4\cdot 2$ → not reverse when multiply by a positive

$-2<8$

$-4<-1$

$$\underbrace{(-4)\cdot(-3)}_{12} > \underbrace{(-1)\cdot(-3)}_{3}$$

reverse the direction when multiply by a negative number