

incorrect words

borrowing

carrying

column shifting

Algorithms

Activity 3

Math-T101 Spring 2014

correct words

rebundling/unbundling

composing/decomposing

regrouping/ungrouping

Name:

Serife Jeviz

Problem 1. Use chip model to solve the following base eight arithmetic problems. Remember that  $(1000)_8$  is called octand.

(a)  $(3420)_8 + (275)_8$

$(1000)_8$ octand	$(100)_8$ octred	$(10)_8$ oct	$(1)_8$ one
000	0000 00 0	00 0000 000 0	0000 0

( 3 | 7 | 1 | 5 )<sub>8</sub>

(b)  $(654)_8 + (677)_8$

octand = $(1000)_8$	octred = $(100)_8$	oct = $(10)_8$	one = $(1)_8$
000000 000000	00000 000000	00000 0000000	00000
0 0 0	0 0 0	0 0 0	0

( 1 | 5 | 5 | 4 )<sub>8</sub>

(c)  $(1334)_8 - (127)_8$

octand = $(1000)_8$	octred = $(100)_8$	oct = $(10)_8$	one = $(1)_8$
0 000	000 0000	000 00000	0000
		000 00000	0000 00000

( 1 | 2 | 0 | 5 )<sub>8</sub>

(d)  $(2062)_8 - (73)_8$

octand = $(1000)_8$	octred = $(100)_8$	oct = $(10)_8$	one = $(1)_8$
000 0000	000 0000	000 00000	00 0000
0 000 000 0000	0 000 000 0000	0 000 0000 00000	00 0000 00000

( 1 | 7 | 6 | 7 )<sub>8</sub>

**Problem 2.** Use lattice addition algorithm to find the answer to the following arithmetic problems.

(a)  $78926 + 9678$

7	8	9	2	6	
	9	6	7	8	
+					
0	1	1	0	1	→ octs/tens ones
7	7	5	9	4	
8	8	6	0	4	↙

(b)  $(3420)_8 + (275)_8$

(	3	4	2	0	)	$_8$
(	2	7	5	)	$_8$	
+					→ octs ones	
0	0	1	0	5		
3	3	6	1	1	5	
3	7	1	5	5	→ $(3715)_8$	

(c)  $(604)_8 + (477)_8$

(	6	0	4	)	$_8$
(	4	7	7	)	$_8$
+					
1	0	1	1	3	
1	3	0	3	3	→ $(1303)_8$

**Problem 3.** Use chip model to find the answer to the following arithmetic problems.

(a)  $14 \times 3$

Tens	Ones
[0]	0000
[0]	0000
[0]	0000
4	2

(b)  $14 \times 10$

Hundreds	Tens	Ones
0	0000	0000
1	4	0

(c)  $14 \times 20$

Hundreds	Tens	Ones
0	0000	0000
2	8	0

(d)  $14 \times 23$

Hundreds	Tens	Ones
0	0000	00
3	2	2

**Problem 4.** Use chip model and the step-by-step method in the previous problem, to find  $(14)_8 \times (23)_8$

(a)  $(14)_8 \times (3)_8$

octs	ones
0	0000
0	0000
0	0000
4	4

b)  $(14)_8 \times (10)_8$

octreds	octs	ones
0	0000	0000
1	4	0
(14) <sub>8</sub>		

c)  $(14)_8 \times (20)_8$

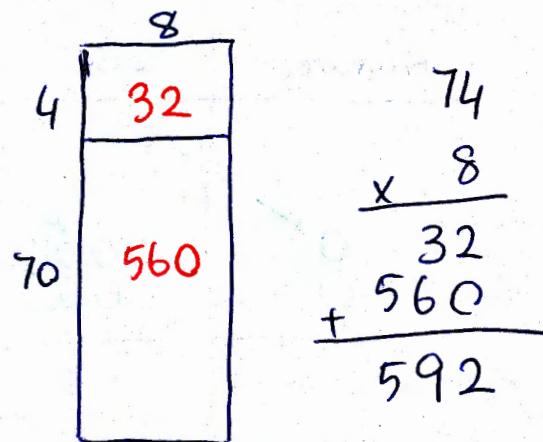
octreds	octs	ones
0	0000	0000
3	0	0
(30) <sub>8</sub>		

d)  $(14)_8 \times (23)_8$

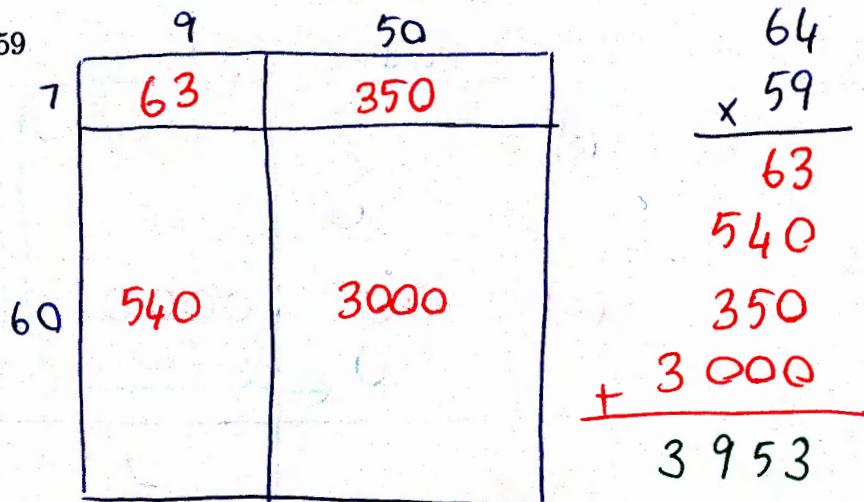
octreds	octs	ones
3	0000	0000
(34) <sub>8</sub>	4	4

**Problem 5.** Use the array model to find the answer to the following arithmetic problems and display the parallel to the SCA for multiplication.

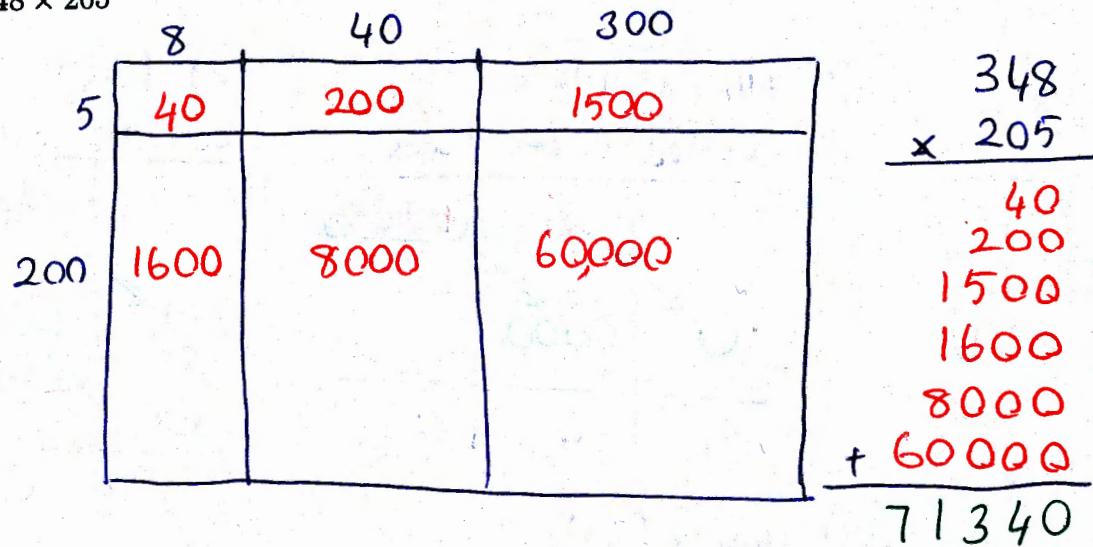
(a)  $74 \times 8$



(b)  $67 \times 59$

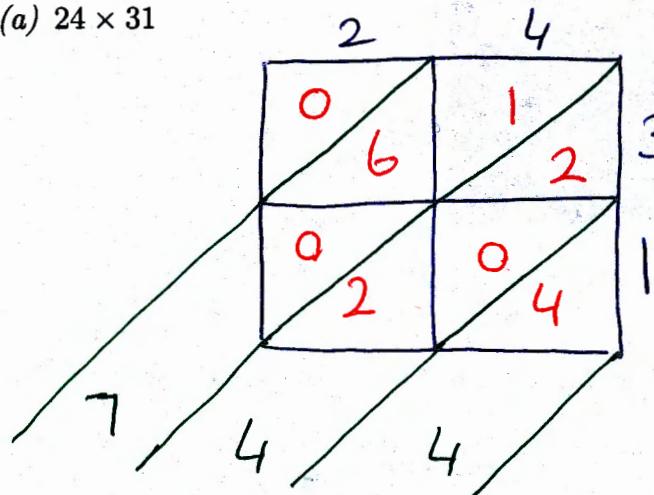


(c)  $348 \times 205$



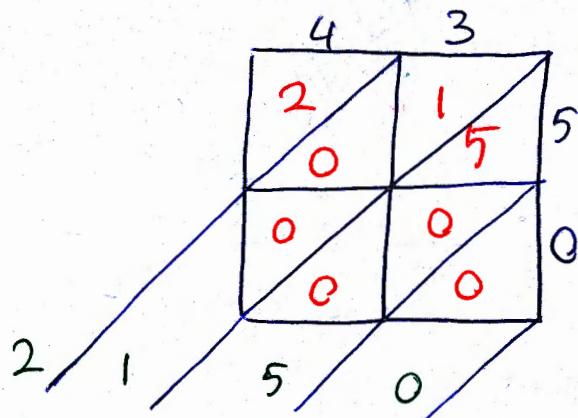
**Problem 6.** Use the lattice algorithm to find the answer to the following arithmetic problems.

(a)  $24 \times 31$



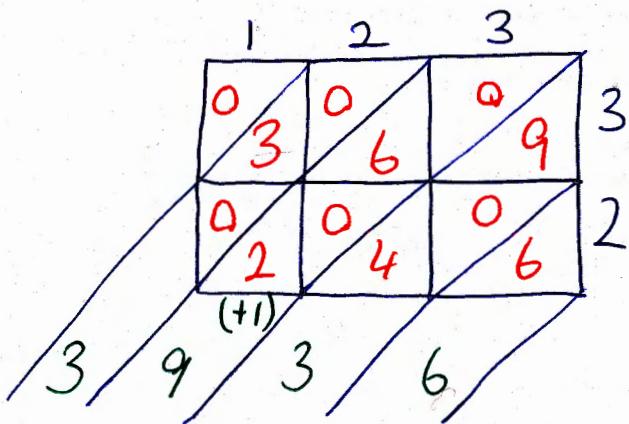
$$\begin{array}{r}
 24 \\
 \times 31 \\
 \hline
 4 \\
 20 \\
 120 \\
 + 600 \\
 \hline
 744
 \end{array}$$

(b)  $43 \times 50$



$$\begin{array}{r}
 43 \\
 \times 50 \\
 \hline
 150 \\
 + 2000 \\
 \hline
 2150
 \end{array}$$

(c)  $123 \times 32$



$$\begin{array}{r}
 123 \\
 \times 32 \\
 \hline
 6 \\
 40 \\
 200 \\
 90 \\
 600 \\
 + 3000 \\
 \hline
 3936
 \end{array}$$

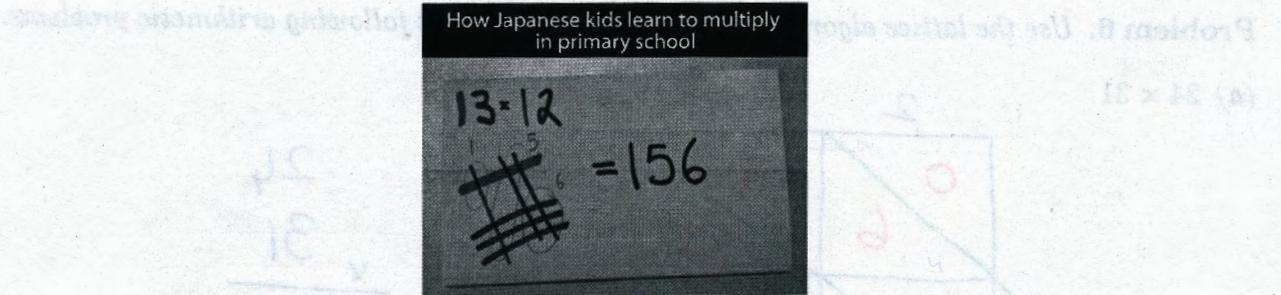
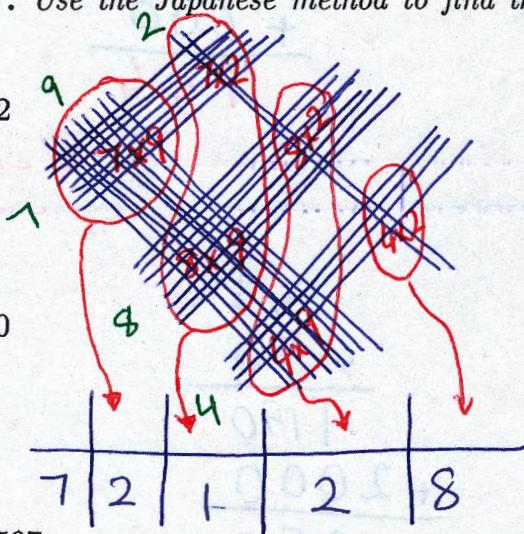


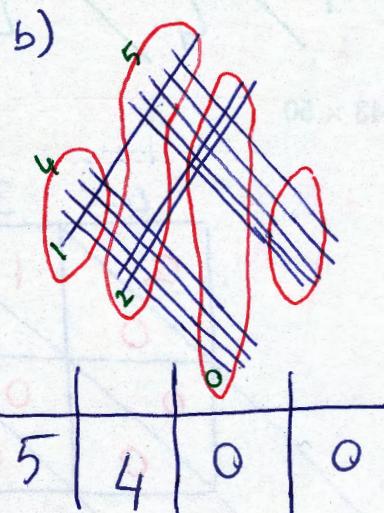
Figure 1: Japanese Multiplication

**Problem 7.** Use the Japanese method to find the answer to the following multiplication problems.

$$(a) 784 \times 92$$



$$(b) 45 \times 120$$



$$(c) 6335 \times 597$$

I will leave this one for students to try.  
It is really long and cumbersome

(d) What are the advantages and limitations of using the above method for multiplication?

- a visual representation of multiplication
- alternative method for the ones who don't remember SCA
- easy to make mistake
- cumbersome
- complicated

**Problem 8.** Use the chip model to evaluate  $(1364)_8 \div (3)_8$ . What interpretation of division did you use? (measurement or partitive)

octonds	Octreds	octs	ones
<input checked="" type="checkbox"/>	000 000 000	0000 00 0000 0000 0000 0000 0000	0000 0000
	000 000 000	00000000 00000000 00000000	0000 0000

partitive { sharing equally among 3 people

Each person will get  $(374)_8$

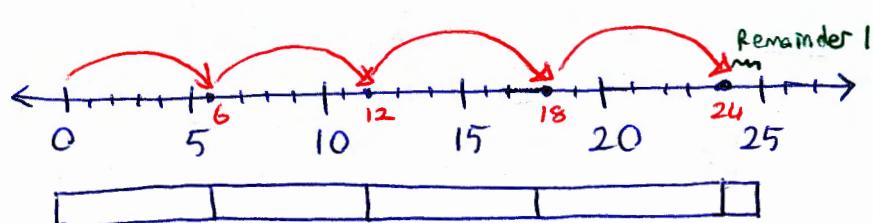
**Problem 9.** Do the previous problem using the other interpretation that you did not use there.

octonds	Octreds	octs	ones
<input type="checkbox"/>	000 000 000	0000 00 0000 0000 0000 0000 0000	0000 0000
	000 000 000	00000000 00000000 00000000	0000 0000

measurement { repeatedly subtract group of 3

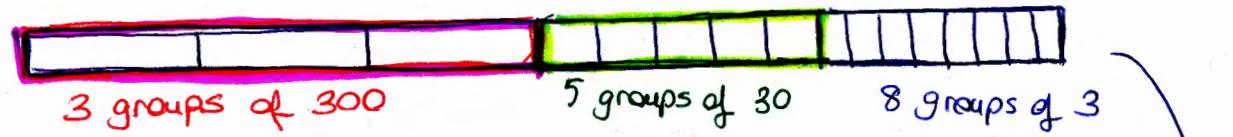
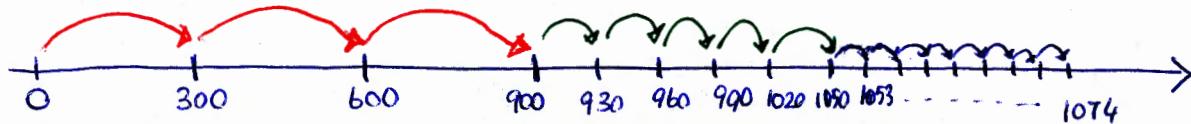
$(3)_8$  groups of  $(300)_8$        $(7)_8$  groups of  $(30)_8$        $(4)_8$  groups of  $(3)_8$        $\rightarrow (374)_8$

**Problem 10.** Use the number line and the measurement interpretation to calculate  $25 \div 6$ .

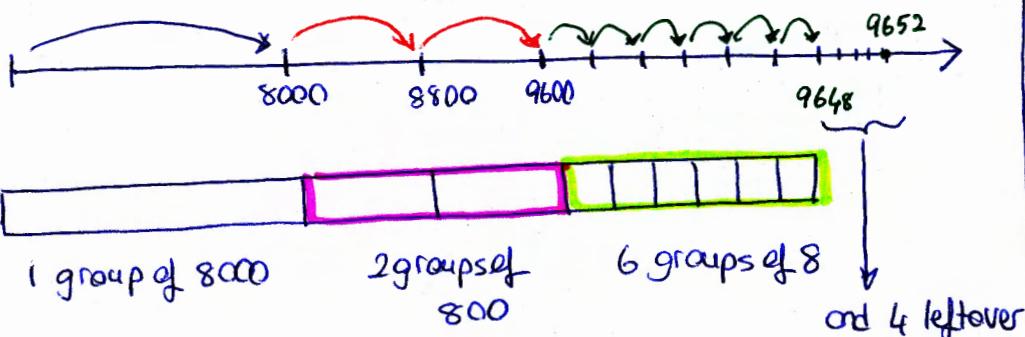


$$\begin{array}{r} 4 R1 \\ 6 \overline{) 25} \\ - 24 \\ \hline 1 \end{array}$$

**Problem 11.** Use measurement division to find  $1074 \div 3$ , laying out segments of lengths 3, 30, and 300 along the number line (you do not have to draw it to scale here). Draw the connection between this method and the SCA for long division.



**Problem 12.** Use measurement division to find  $9652 \div 8$ , laying out segments of lengths 8, 80, and 800, etc. along the number line (you do not have to draw it to scale here). Draw the connection between this method and the SCA for long division.



$$\begin{array}{r} 1206 R4 \\ 8 \overline{) 9652} \end{array}$$

$$\begin{array}{r} \\ - 8000 \\ \hline 1652 \end{array} \rightarrow 1 \text{ group of length 8000}$$

$$\begin{array}{r} \\ - 1600 \\ \hline 52 \end{array} \rightarrow 2 \text{ groups of length 800}$$

$$\begin{array}{r} \\ - 48 \\ \hline 04 \end{array} \rightarrow 6 \text{ groups of 8}$$

$$\begin{array}{r} \\ - 4 \\ \hline 0 \end{array} \rightarrow 4 \text{ leftover}$$

$$\begin{array}{r} 358 \\ 3 \overline{) 1074} \\ - 900 \\ \hline 174 \\ - 150 \\ \hline 24 \\ - 24 \\ \hline 0 \end{array}$$

→ 3 groups of length 300  
 → 174 leftover  
 → 5 groups of length 30  
 → 24 leftover  
 → 8 groups of length 3  
 → no leftover

estimation < actual answer

Problem 13. Underestimate  $(1647)_8 - (376)_8$  by rounding the minuend and the subtrahend to multiples of octred. (See p. 19 for the definitions of these terms.) Which number would you round up? Which number would you round down? Is your estimate correct to the nearest octred?

$$\underbrace{(1647)_8}_{\text{minuend}} - \underbrace{(376)_8}_{\text{subtrahend}}$$

actual answer

$$\begin{array}{r} (1647)_8 \\ - (376)_8 \\ \hline (1251)_8 \end{array}$$

$\downarrow \quad \uparrow$

$$\text{or/} \quad (1600)_8 - (400)_8 = (1200)_8$$

$$(1640)_8 - (400)_8 = (1240)_8 \rightarrow \text{better estimate}$$

Problem 14. How would you overestimate  $1345 \div 72$ ? (Would you round 72 up or down?)  
How about 1345?

↑  
up      estimation > actual result      ↓

$$1400 \div 70 = 20$$

actual result

$$\begin{array}{r} 18 \text{ R } 49 \\ 72 \overline{)1345} \\ - 72 \\ \hline 625 \\ - 576 \\ \hline 49 \end{array}$$

Problem 15. Use long division to find  $1455 \div 15$ .

$$15 \overline{)1455}$$

q <sup>9</sup> <sub>7</sub>  
 $135$   
 $\underline{- 105}$   
 $= 105$   
 $\underline{- 105}$   
 $= 00$

$15 \times 10 = 150$   
 $\underline{- 15}$   
 $135$

$15 \times 6 = 90$   
 $+ 15$   
 $105$

**Problem 16.** Use long division to find  $27003 \div 45$ .

$$\begin{array}{r} 600 \text{ R } 3 \\ 45 \sqrt{27003} \\ - 270 \\ \hline 00003 \end{array}$$

$$45 \times 2 = 90 \times 3 = 270$$

**Problem 17.** Use long division to find  $14985 \div 37$ .

$$\begin{array}{r} 405 \\ 37 \sqrt{14985} \\ - 148 \\ \hline 185 \\ - 185 \\ \hline 00 \end{array}$$

$$\begin{array}{r} 37 \times 2 = 74 \times 2 = 148 \\ + 37 \\ \hline 185 \end{array}$$