INFO I201
Sample Midterm I

Assigned 05/20. Not to be collected.

1. (a) Use truth trees to show that

$$\neg P \vee \neg Q \equiv Q \longrightarrow \neg P$$

   (b) Is $(A \longrightarrow (B \longrightarrow C)) \longrightarrow ((A \longrightarrow B) \longrightarrow (A \longrightarrow C))$ a tautology? Either show it is, using any method that we discussed in class, or give a counterexample in case the formula is not a tautology.

   (c) Is $(((Q \wedge R) \longrightarrow R) \wedge (R \longrightarrow P)) \longrightarrow Q$ satisfiable?

2. Give natural deduction proofs for the following sequents.

   (i) $(A \wedge B) \longrightarrow C \vdash A \longrightarrow (B \longrightarrow C)$

   (ii) $A, B \vee C \vdash (A \wedge B) \vee (A \wedge C)$

   (iii) $A \vee (B \wedge C), \neg A \vdash C$

   (iv) $A \wedge B \vdash A \vee B$

3. Are the following statements consistent?

   "When the mixture begins to bubble, there will be a delicious aroma of vanilla beans, if the ingredients are unspoiled. As the mixture begins to cool, there will be a faint red striation across the surface of the liquid. Either the mixture cools or there will be a delicious aroma of vanilla beans. " (use B, A, S, C, R).

4. Consider the following argument:

   "If the rudder does not break and the fuel holds out, the ship will get safely to port and no one will drown. If the fuel holds out but the rudder breaks, then the ship can be stressed by means of its propellers, and if so, then it will get safely to port. If the rudder does not break, the fuel will hold out. Therefore, the ship will get safely to port." (use R, F, P, D, S.)

   Is this argument valid? If yes, give a proof (using any method that we discussed in class), if not give a counterexample.