S520 Homework 6

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9.6.#1: H_0 : John Spriko is innocent vs. H_1 : John Spriko is not innocent.
Thus, the decision rule is: $\mathbf{p} \leq \alpha$ then declare Spirko guilty, where $\mathbf{p}$ is a number quantifying the evidence against Spriko and α the significance level. In light of the fact that Spriko was sentenced, we know that the former decision hold true. Also, we known that α is the probability of committing a Type I error. In this case, there is a 30% chance that the witness made a mistake identifying Spirko. Thus, $\alpha = 30\%$.
It seems to me that this level of significance is not suitable for a capital murder trial. In general, if we allow this significance level, then on average the system will sentence 3 innocent people to death out of 10 being judged. That is not an acceptable error.

9.6.#4:

(a) H_0 : evaporated milk contain less than or equal to 23% by weight of total milk vs
H_1 : evaporated milk contain more than 23% by weight of total milk.
The reason is that the consumers want the burden of proof on the side of the company. They also want to minimize Type 1 error, which in this case would mean that the milk is in fact not FDA compliant but the outcome of the test shows otherwise.

(b) H_0 : evaporated milk contain more or equal to 23% by weight of total milk vs
H_1 : evaporated milk contain less than 23% by weight of total milk
The reason is that the company want the burden of proof on the side of the consumers. They also want to minimize Type 1 error, which in this case would mean that the milk is in fact FDA compliant but the outcome of the test shows otherwise. In this case the company would have to pay unnecessary damages.

(c) Given that $n = 225$, $\bar{x} = 22.8$ and $s = 3$ we can calulate $t = \frac{22.8-23}{3/\sqrt{225}} = \frac{-0.2}{0.2} = -1$

(d) $\mathbf{p} = P(T_n \geq t) = 1 - P(T_n \leq -1) = 1 - pnorm(-1) = 0.8413$. The decision rule states:

$$\mathbf{p} = 0.8414 > 0.05 = \alpha \Longrightarrow \text{ decline to reject } H_0$$

(e) $\mathbf{p} = P(T_n \leq t) = P(T_n \leq -1) = pnorm(-1) = 0.1587$. The decision rule states:

$$\mathbf{p} = 0.1587 > 0.05 = \alpha \Longrightarrow \text{ decline to reject } H_0$$

9.6.#6: $H_0 : \mu \geq 800$ vs. $\mu < 800$.
I assume that the company would not want to waste unnecessary funds, and that they are going to counter advertise the alleged exaggerated claims of the light bulb manufacturer only if there is sufficient evidence to do so. In short, they want to minimize Type I error, which in this case would be that the bulbs actually work as advertised but that the test result support the opposite statement.

Let $\alpha = 0.05$, then $\mathbf{p} = P(T_n < t)$, where $t = \frac{\bar{x}-\mu}{s/\sqrt(n)} = \frac{745.1-800}{238/\sqrt(100)} = \frac{-54.9}{23.8} = -2.3067$. Thus, $P(T_n < -2.3067) = 0.0105$. The decision rule states:

$$\mathbf{p} = 0.0105 \leq 0.05 = \alpha \Longrightarrow \text{ reject } H_0$$

9.6.#7: $H_0 : \mu = 0$ vs. $\mu \neq 0$.
This setup is appropriate because we want to test whether or not there is enough evidence to claim that showing the different pictures at different times has any effect. The Type I error here is that there is indeed a difference and the test shows otherwise. This is exactly the type of error the study should minimize.

$t = \frac{\bar{x}-\mu}{s/\sqrt(n)} = \frac{-0.1833}{5.18633/\sqrt(60)} = \frac{-0.1833}{0.6696} = -0.2737$. Now, $\mathbf{p} = P(|T_n| \geq |t|) = 2 * pnorm(-|t|) = 0.7842$.

The decision rule states:

$$\mathbf{p} = 0.7842 > 0.05 = \alpha \Longrightarrow \text{ decline to reject } H_0$$

9.6.#9: In this case, it is true that $1 - \alpha = 0.99 \Longrightarrow \alpha = 0.01$ and thus, $q = qnorm(1 - (0.01/2)) = 2.5758$

$$n = (2q\sigma/L)^2. \text{ If } L = 2, \sigma = 6 \text{ and } q = 2.5758 \text{ then } n = (2 * 2.5758 * 6)/2 \approx 238.8563$$

SAHC should plan to take $n = 239$ measurements.

9.6.#11: To obtain the number of evaluations for a length L, we can use the following fact:

$$n = (2q\sigma/L)^2$$

From the errata, we know that $L = 0.001$. q can be obtained as follows: $1 - \alpha = 0.95 \Longrightarrow \alpha = 0.05$, thus $q_{1-\frac{\alpha}{2}} = q_{1-\frac{0.05}{2}} = q_{1-0.025} = q_{0.975} = qnorm(0.975) = 1.9600$. At this point we are only missing σ. We need to estimate this quantity and we can model the situation like this: $X_1, X_2, ..., X_n \sim Bernoulli(p)$, where each $X_i = 1$ if the observed value lies between 170.5 and 199.5 and $X_i = 0$ otherwise. By properties of a Bernoulli trial, $EX_i = p$ and $VarX_i = p(1-p)$. The parameter p can be approximated using the CLT, in fact we did this for the last homework. I repeat the calculations here:

$\mu = EV_i = 0.4 + 0.2 + 1 + 3 = 4.6$, where V is a result of taking a number out of the urn,
$\sigma^2 = VarV_i = EV_i^2 - (EV_i)^2 = 0.4 + 0.4 + 5 + 30 - (4.6)^2 = 35.8 - 21.16 = 14.64$,

By the CLT, $Y \approx Normal(n\mu, n\sigma^2) = Normal(184, \sqrt{585.6})$, and thus,

$$P(170.5 < Y < 199.5) = P(Y \leq 199.5) - P(Y \leq 170.5) = pnorm(199.5, 184, \sqrt{585.6}) - pnorm(170.5, 184, \sqrt{585.6})$$
$$= 0.4506 = p$$

Now, we can calculate $VarX_i = p(1-p) = 0.4506(1 - 0.4506) = 0.2476 \Longrightarrow \sigma = \sqrt{(}0.24756) = 0.4976$.
Finally,

$$n = (2q\sigma/L)^2 = (2 * 1.9600 * 0.4976/0.001)^2 = 3,804,809$$