M463 Homework 6

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(2.1) #6 A man fires 8 shots at a target. Assume that the shots are independent, and each shot hits the bull's eye with probability 0.7.

a) What is the chance that he hits the bull's eye exactly 4 times?

Solution: Since these are independent trials we can use the binomial distribution. Let $n = 8$ be the number of trials. Call a success if the man hits the bull's eye with $P(success) = 0.7$. Then, the probability that he hits the bull's eye exactly 4 times is given by:

$$P(\text{4 successes in 8 trials}) = \binom{8}{4} 0.7^4 0.3^4 = \boxed{0.1361367}$$

b) Given that he hit the bull's eye at least twice, what is the chance that he hit the bull's eye exactly 4 times?

Solution: This is a conditional probability. Hence,

$$P(\text{4 successes in 8 trials} \mid \text{at least 2 successes}) = \frac{P(\text{4 successes in 8 trials } \cap \text{ at least 2 successes})}{P(\text{ at least 2 successes})}$$

But the intersection in the numerator is just the event of 4 successes in 8 trials, since if you have exactly 4 successes then certainly you had at least 2. Hence,

$$\frac{P(\text{4 successes in 8 trials } \cap \text{at least 2 successes})}{P(\text{at least 2 successes})} = \frac{P(\text{4 successes in 8 trials })}{P(\text{at least 2 successes})}$$

The numerator was previously computed in part a). The denominator is easier calculated with the complement rule:

$$P(\text{at least 2 successes}) = 1 - P(\text{zero OR one success in 8 trials}) = 1 - \binom{8}{0} 0.7^0 0.3^8 - \binom{8}{1} 0.7^1 0.3^7 = 0.99870967$$

Finally, the desired probability is:

$$P(\text{4 successes in 8 trials} \mid \text{at least 2 successes}) = \frac{0.1361367}{0.99870967} = \boxed{0.13631259}$$

c) Given that the first two shots hit the bull's eye, what is the chance that he hits the bull's eye exactly 4 times in the 8 shoots?

Solution: Since two of the shoots were already made and shots are independent, we can think of this probability as the probability of hitting exactly 2 out of the remaining 6 shoots:

$$P(\text{4 successes in 8 trials} \mid \text{the first 2 were successes}) = P(\text{2 successes in 6 trials}) = \binom{6}{2} 0.7^2 0.3^4 = \boxed{0.59535}$$