M463 Homework 4

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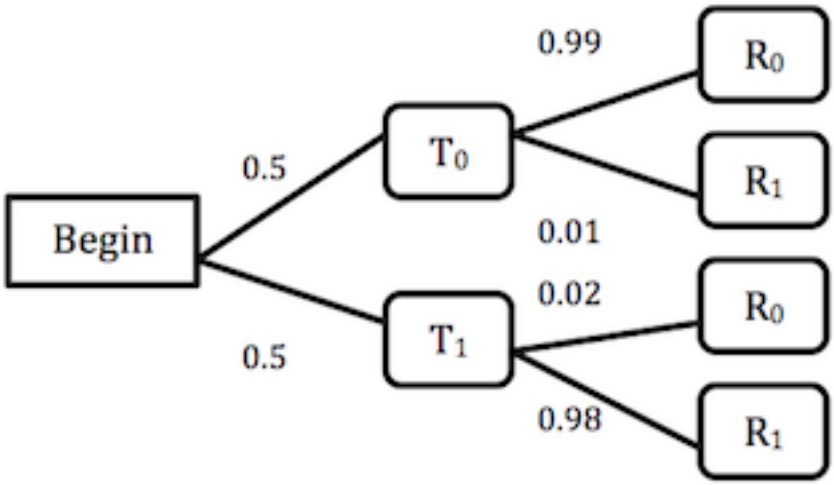
(1.5) #4 Consider the following events:

$T_0 = \{\text{Transmitter sends 0}\}$, $R_0 = \{\text{Receiver concludes that a 0 was sent}\}$
$T_1 = \{\text{Transmitter sends 1}\}$, $R_1 = \{\text{Receiver concludes that a 1 was sent}\}$

Assume that $P(R_0|T_0) = 0.99, P(R_1|T_1) = 0.98$, and $P(T_1) = 0.5$. Then:



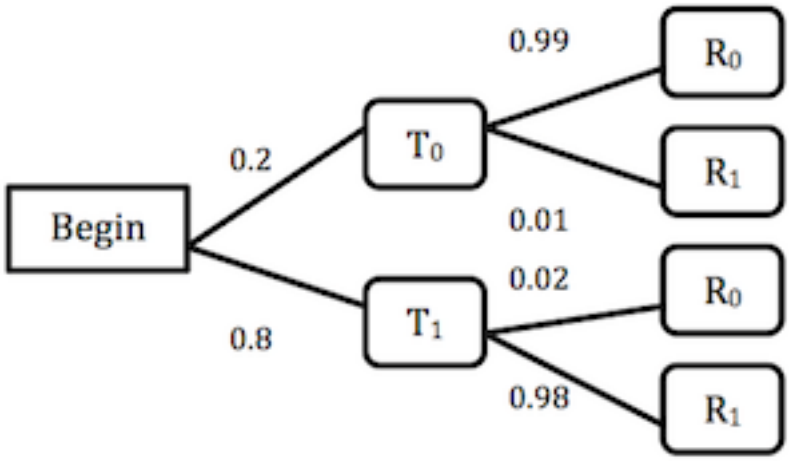
a) The probability of a transmission error given R_1 is:

$$P(T_0|R_1) = \frac{P(T_0 \cap R_1)}{P(R_1)} = \frac{P(T_0 \cap R_1)}{P((T_0 \cap R_1) \cup (T_1 \cap R_1))} = \frac{0.005}{0.005 + 0.49} = \boxed{\frac{0.005}{0.495} = 0.010101}$$

b) The overall probability of a transmission error is:

$$P((R_0 \cap T_1) \cup (R_1 \cap T_0)) = P(T_0) \cdot P(R_1|T_0) + P(T_1) \cdot P(R_0|T_1) = 0.5 \cdot 0.01 + 0.5 \cdot 0.02 = \boxed{0.015}$$

Now, assume that $P(R_0|T_0) = 0.99, P(R_1|T_1) = 0.98$, and $P(T_1) = 0.8$. Then:



a) The probability of a transmission error given R_1 is:

$$P(T_0|R_1) = \frac{P(T_0 \cap R_1)}{P(R_1)} = \frac{P(T_0 \cap R_1)}{P((T_0 \cap R_1) \cup (T_1 \cap R_1))} = \frac{0.002}{0.002 + 0.784} = \boxed{\frac{0.002}{0.786} = 0.002544}$$

b) The overall probability of a transmission error is:

$$P((R_0 \cap T_1) \cup (R_1 \cap T_0)) = P(T_0) \cdot P(R_1|T_0) + P(T_1) \cdot P(R_0|T_1) = 0.2 \cdot 0.01 + 0.8 \cdot 0.02 = \boxed{0.018}$$