M463 Homework 18

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July 22, 2013

For random variables X and Y with joint density function

$$f(x, y) = 6e^{-2x-3y} \qquad (x, y > 0)$$

and $f(x, y) = 0$ otherwise, find:

a) $P(X \leq x, Y \leq y)$;

Solution:

$$F(x,y) = \int_0^y \int_0^x f(x,y)dxdy = \int_0^y \int_0^x 6e^{-2x-3y}dxdy = 6\int_0^y e^{-3y}dy \int_0^x e^{-2x}dx = 6\left(\left[-\frac{e^{-3y}}{3}\right]_0^y \left[-\frac{e^{-2x}}{2}\right]_0^x\right)$$

$$= 6\left(-\frac{e^{-3y}}{3} + \frac{1}{3}\right)\left(-\frac{e^{-2x}}{2} + \frac{1}{2}\right) = \boxed{(1 - e^{-3y})(1 - e^{-2x})}$$

This could also be answered by noticing that these are independent variables (see part d)), and then multiply the corresponding cdfs of the exponential distributions:

$$P(X \leq x, Y \leq y) = F_X(x)F_Y(y) = \boxed{(1 - e^{-3y})(1 - e^{-2x})}$$

b) $f_X(x)$;

Solution:

$$f_X(x) = \int_0^\infty f(x,y)dy = \int_0^\infty 6e^{-2x-3y}dy = 6e^{-2x}\left[-\frac{e^{-3y}}{3}\right]_0^\infty = 6e^{-2x}\left(0 + \frac{1}{3}\right) = \boxed{2e^{-2x}}$$

c) $F_Y(y)$.

Solution:

$$f_Y(y) = \int_0^\infty f(x,y)dx = \int_0^\infty 6e^{-2x-3y}dy = 6e^{-3y}\left[-\frac{e^{-2x}}{2}\right]_0^\infty = 6e^{-3y}\left(0 + \frac{1}{2}\right) = \boxed{3e^{-3x}}$$

d) Are X and Y independent? Give a reason for your answer.

Solution: Yes, they are independent since for every x, y:

$$\boxed{f(x,y) = 6e^{-2x-3y} = 2e^{-2x} \cdot 3e^{-3x} = f_X(x)f_Y(y)}$$