M463 Homework 16

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A Geiger counter is recording background radiation at an average rate of one hit per minute. Let T_3 be the time in minutes when the third hit occurs after the counter is switch on. Find $P(2 \leq T_3 \leq 4)$.

Solution: T has Gamma distribution with parameters $\lambda = \dfrac{1 \text{ hit}}{\text{min}}$ and $r = 3$. Therefore:

$$P(2 \leq T_3 \leq 4) \quad = \quad \int_2^4 \frac{e^{-t}t^2}{2!}dt \quad = \quad \frac{1}{2}\int_2^4 e^{-t}t^2 dt$$

Applying Integration by parts twice to : $\int e^{-t}t^2 dt$

$$\int e^{-t}t^2 dt \quad = \quad 2\int e^{-t}t dt - t^2e^{-t} \quad = \quad 2(-e^{-t} - te^{-t}) - t^2e^{-t}$$

Therefore, since : $\frac{1}{2}\int e^{-t}t^2 dt = -\frac{1}{2}[e^{-t}t^2 + 2te^{-t} + 2e^{-t}]$

$$\begin{aligned}
P(2 \leq T_3 \leq 4) \quad = \quad -\frac{1}{2}\left[e^{-t}t^2 + 2te^{-t} + 2e^{-t}\right]_2^4 \quad &= \quad -\frac{1}{2}\left[(16e^{-4} + 8e^{-4} + 2e^{-4}) - (4e^{-2} + 4e^{-2} + 2e^{-2})\right] \\
&= \quad -\frac{1}{2}\left[26e^{-4} - 10e^{-2}\right] \\
&= \quad 5e^{-2} - 13e^{-4} \\
&= \quad \boxed{0.43857}
\end{aligned}$$