M463 Homework 13

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On average, one cubic inch of Granma's cookie dough contains 2 chocolate chips and 1 marshmallow.

a) Granma makes a cookie using three cubic inches of her dough. Find the chance that the cookie contains at most four chocolate chips. State your assumptions.

Solution: Suppose that no two chocolate chips are in exactly the same point and that each point has the same (small) probability of having a chocolate chip. Let X = number of chocolate chips on a cookie made using 3 cubic inches of dough. Then

$$X \sim Pois(\mu = \frac{2 \text{ choc. chips}}{1 \text{ inch}^3} \cdot 3 \text{ inch}^3 = 6)$$

Hence, $P(X \leq 4) = P(X = 0 \text{ OR } X = 1 \text{ OR } X = 2 \text{ OR } X = 3 \text{ OR } X = 4) = e^{-6}\left(\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!} + \frac{6^4}{4!} +\right) =$ $\boxed{115e^{-6} = 0.29}$

b) Assume the number of marshmallows in Granma's dough is independent of the number of chocolate chips. I take three cookies, one of which is made with two cubic inches of dough, the other two with three cubic inches each. What is the chance that at most 1 of my cookies contains neither chocolate chips nor marshmallows?

Solution: Let X_{ci} = number of chocolate chips on cookie i. Likewise, let X_{mi} = number of marshmallows on cookie i. Then, $X_{c1} \sim Pois(4)$ and $X_{c2}, X_{c3} \sim Pois(6)$. Also, $X_{m1} \sim Pois(2)$ and $X_{m2}, X_{m3} \sim Pois(3)$. Finally, let $Y_i = X_{ci} + X_{mi}$ be the number of chocolate chips and marshmallows in cookie i. Since the sum of independent Poisson variables is also Poisson we have that $Y_1 \sim Pois(6)$ and $Y_2, Y_3 \sim Pois(9)$. We wish to know the following probability:

$$P(\text{at most 1 of my cookies contains neither chocolate chips nor marshmallows}) =$$

$P(Y_1 = 0, Y_2 > 0, Y_3 > 0) + P(Y_1 > 0, Y_2 = 0, Y_3 > 0) + P(Y_1 > 0, Y_2 > 0, Y_3 = 0) + P(Y_1 > 0, Y_2 > 0, Y_3 > 0) =$

$P(Y_1 = 0)P(Y_2 > 0)P(Y_3 > 0) + P(Y_1 = 0)P(Y_2 > 0)P(Y_3 > 0) + P(Y_1 = 0)P(Y_2 > 0)P(Y_3 > 0) + P(Y_1 = 0)P(Y_2 > 0)P(Y_3 > 0) =$

$$e^{-6}(1 - e^{-9})^2 + (1 - e^{-6})e^{-9}(1 - e^{-9}) + (1 - e^{-6})(1 - e^{-9})e^{-9} + (1 - e^{-6})(1 - e^{-9})^2 =$$

$$\boxed{(1 - e^{-9})^2 + 2(1 - e^{-6})(1 - e^{-9})e^{-9}}$$