M451/551 Quiz 1

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1. A lawyer must decide whether to charge a fixed fee of $5,000 or take a contingency fee of $25,000 if she wins the case (and 0 if she loses). She estimates that her probability of winning is .30. Determine the mean and standard deviation of her fee if

   (a) she takes the fixed fee;

   (b) she takes the contingency fee.

(a) If she takes the fixed fee, then there is no randomness involved, which means that the mean is just \$5,000 and the standard deviation is zero. Another -rather silly- way is to let X = her fee. And then $P(X = \$5{,}000) = 1$. then $E[X] = 1 \cdot \$5{,}000 = \$5{,}000$.
$\text{Var}(X) = E[(X - E[X])^2] = E[(5{,}000 - 5{,}000)^2] = E[0] = 0 \Rightarrow \text{STdev} = \sqrt{0} = 0$. ✓

(b) In this case, let X = her fee. then, the following table summarizes the data:

X	$P(X=x)$	$x \cdot P$
0	7/10	$0 \cdot 7/10 = 0$
25,000	3/10	$25{,}000 \cdot \frac{3}{10} = 2{,}500 \times 3 = 7{,}500$

which means, $E[X] = 0 + 7{,}500 = \$7{,}500$.

The variance is given by: $E[(X - E[X])^2] = (0 - 7{,}500)^2 \cdot \frac{7}{10} + (25{,}000 - 7{,}500)^2 \cdot \frac{3}{10}$

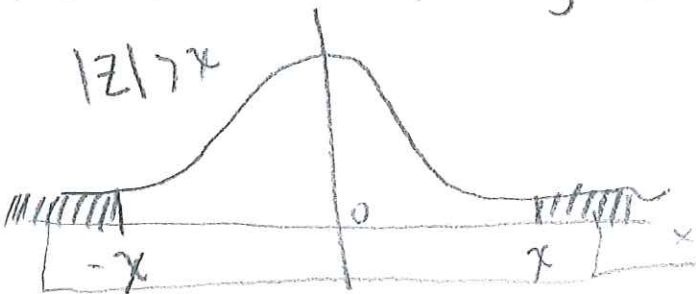
$= (7{,}500)^2 \cdot \frac{7}{10} + (17{,}500)^2 \cdot \frac{3}{10} = \frac{75^2 \times 10^2 \times 7}{10} + \frac{175^2 \times 10^2 \times 3}{10} =$

$= 75^2 \times 10 \times 7 + 175^2 \times 10 \times 3$, the standard deviation is the square of this quantity $\sigma = \sqrt{75^2 \times 10 \times 7 + 175^2 \times 10 \times 3}$ +10

2. Argue that $P\{|Z| > x\} = 2P\{Z > x\}$, where $x > 0$ and Z is a standard normal random variable.

the event $|Z| > x$ means that $Z < -x$ or $Z > x$, for $x > 0$

Since the standard normal random variable is symmetric, it follows that $P\{|Z| > x\} = 2P\{Z > x\}$, since probability is the area under the curve. the following picture depicts this situation.



Both areas in the picture are the same, hence:

$P\{|Z| > x\} = P\{Z > x\} + P\{Z > x\}$

$= 2P\{Z > x\}$ +10