M447 - Mathematical Models/Applications 1 - Homework 6

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Chapter 3, Section 3.4

(7) Consider a small-group decision-making situation similar to that described in Section 3.1 with six individuals and three alternatives. Formulate a Markov chain model under the following assumption: An individual who is the only person voting for an alternative is twice as likely to change her vote as a person who is one of a group of two or more voting for an alternative. If an individual changes her vote, then the probability of changing to an alternative is proportional to the number of individuals voting for that alternative.

(a) If the group is initially divided 3,2, and 1, find the expected number of vote changes before consensus is reached.

Solution: The possible states of the Markov Chain are $600, 510, 420, 411, 321, 330, 222$, where 600 is an absorbing state corresponding to consensus. The matrix of transition probabilities is given by:

P =

	600	510	420	411	321	330	222
600	1	0	0	0	0	0	0
510	2/7	0	5/7	0	0	0	0
420	0	2/6	0	0	0	4/6	0
411	0	2/5	2/20	0	1/2	0	0
321	0	0	6/35	3/14	5/14	4/35	1/7
330	0	0	1	0	0	0	0
222	0	0	0	0	1	0	0

This matrix is already in canonical form. Hence, we can get the matrix Q, which is:

Q =

	510	420	411	321	330	222
510	0	5/7	0	0	0	0
420	2/6	0	0	0	4/6	0
411	2/5	2/20	0	1/2	0	0
321	0	6/35	3/14	5/14	4/35	1/7
330	0	1	0	0	0	0
222	0	0	0	1	0	0

Assuming we have this matrix as a variable named Q in mathematica, we just type the command:

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Inverse[IdentityMatrix[6] - Q]
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to get the fundamental matrix N

N =

	510	420	411	321	330	222
510	7/2	15/2	0	0	5	0
420	7/2	21/2	0	0	7	0
411	7/2	987/110	14/11	14/11	337/55	2/11
321	7/2	1083/110	6/11	28/11	377/55	4/11
330	7/2	21/2	0	0	8	0
222	7/2	1083/110	6/11	28/11	377/55	15/11

Consequently, the expected number of vote changes before consensus is reached when the initial group is divided 321 is the sum of all entries in row 4:

$$\boxed{\frac{7}{2} + \frac{1083}{110} + \frac{6}{11} + \frac{28}{11} + \frac{377}{55} + \frac{4}{11} = \frac{1301}{55}}$$

So we expect about $\frac{1301}{55} \approx 23.6545$ vote changes before consensus.