M447 - Mathematical Models/Applications 1- Homework 1

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Chapter 2, Section 2.5

(5) A ten-stage model for human population growth has the reproduction and survival rates as shown below. Using these data, estimate the long-term rate of increase and the long-term distribution of the population in the ten age groups.

Stage	f_i	s_i
1	0	.997
2	.001	.998
3	.085	.998
4	.306	.997
5	.400	.996
6	.281	.995
7	.153	.992
8	.064	.989
9	.015	.983
10	.001	

Solution: Using these data we can build the following matrix of vital rates for the population:

$$\mathbf{A} = \begin{bmatrix} 0 & .001 & .085 & .306 & .400 & .281 & .153 & .064 & .015 & .001 \\ .997 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .998 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .998 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .997 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .996 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .995 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .992 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .989 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .983 & 0 \end{bmatrix}$$

By fact (2.26), to find the the long-term rate of increase and the long-term distribution of the population in the ten age groups, we need to find the largest eigenvalue and its associated eigenvector (which will be normalized to add up to 1).

I used Octave to find the eigenvalues and eigenvector. In Octave, assuming one has the matrix A in memory, use the commands: $[V, lambda] = eig(A)$. The largest eigenvalue is approximately $\boxed{\lambda_0 = 1.04995}$, and its corresponding eigenvector is $V_{\lambda_0} = [-0.39146, -0.37172, -0.35333, -0.33585, -0.31891, -0.30252, -0.28669, -0.27087, -0.25515, -0.23888]$.

Finally, let us normalize this vector so that its components add up to one. To that end, let us find the value of x such that:

$$x \cdot \sum_{i=1}^{10} V_{\lambda_0}(i) = 1 \Longrightarrow x = \frac{1}{-0.39146 - 0.37172 - 0.35333 - 0.33585 - 0.31891 - 0.30252 - 0.28669 - 0.27087 - 0.25515 - 0.23888}$$

$$\Longrightarrow x = -0.31996109273$$

Hence, our normalize vector $\mathbf{W} = x \cdot V_{\lambda_0}$ is

$$\boxed{\mathbf{W} = [0.125252, 0.118936, 0.113051, 0.107458, 0.102039, 0.096796, 0.091730, 0.086668, 0.081637, 0.076431]}$$

The long-term rate of increase is λ_0 and the long-term distribution o the population is given by $\mathbf{W}$.