
— M436 — Homework Assignment 8 —

Due: Wednesday, October 29, in class. Each problem is worth 20 points. Please show all your work. Homework that is illegible or discourages the reader otherwise from looking at it will be returned ungraded.

Exercise 1

Find all quaternions $q = a + bi + cj + dk$ such that $q^2 = -13 + 6i - 2j + 4k$.

Exercise 2

Given a spherical triangle Δ with angles $0 < \alpha, \beta, \gamma < \pi$. For which choice of angles is there a tiling of the sphere S^2 by triangles with the same angles as Δ such that neighboring triangles are symmetric with respect to their shared edge?

Exercise 3

In the previous exercise, you should have found an example with $\alpha = \beta = \gamma = 72^\circ$. Consider one of the triangles, and denote the reflections in S^2 at the edges this triangle by ρ, ϕ, ψ . Let G be the subgroup of $O(3)$ generated by these reflections. Draw the Cayley graph of G with respect to the set $\Gamma = \{\rho, \phi, \psi\}$.

Exercise 4

Determine all linear automorphisms of \mathbf{F}_2^n (i.e. invertible $n \times n$ -matrices with entries in \mathbf{F}_2) that are isometries with respect to the Hamming distance.

Exercise 5

Let q be an imaginary unit quaternion, i.e. $|q| = 1$ and $\bar{q} = -q$. Show that the unit quaternions p such that $pq\bar{p} = q$ have the form $p = r + sq$ with $r, s \in \mathbf{R}$ and $r^2 + s^2 = 1$.