Homework - S403

Due September 3, 2013

Remember to write on only one side of the sheet.

1. Let S be a set and let $f : S \to S$ be a function.

(a) Prove that f is one-to-one if and only if there exists a function $g : S \to S$ such that $g \circ f = \ id$.

(b) Prove that f is onto if and only if there exists a function $g : S \to S$ such that $f \circ g =$ id.

(c) Prove that f is one-to-one and onto if and only if there exists a function $g : S \to S$ such that $f \circ g = g \circ f =$ id.

(d) Give an example of a set T with an associative operation with identity e for which there is an element x in T that is "left" invertible (that is, there is an element y such that $yx = e$) but not invertible.

2. Let S be a finite set and let $f : S \to S$ be a function. Prove the following conditions are equivalent:
(a) f is one-to-one.
(b) f is onto.
(c) f is one-to-one and onto.

3. Let $(G, \#)$ be a group and let H be a nonempty finite subset of G. Prove that H is a subgroup of G if and only if H is closed under $\#$.

4. Let G be a set with an associative operation that satisfies the following two properties:

(a) There is an element e in G such that $ge = g$ for all $g \in G$.
(b) For each $g \in G$ there is an element $h \in G$ such that $gh = e$.

Prove that G is a group under this operation.

5. Write down the group table for D_4.

6. Determine the elements of D_5, the group of symmetries of the regular pentagon. You will probably want to follow the sequence of steps we used for D_3 and D_4.