M343 Homework 3

Enrique Areyan
May 17, 2013

Section 2.6

3. Consider the equation: $(3x^2 - 2xy + 2)dx + (6y^2 - x^2 + 3)dy = 0$. Let $M(x,y) = 3x^2 - 2xy + 2$ and $N(x,y) = 6y^2 - x^2 + 3$. Since:

$$\frac{\partial M}{\partial y} = -2x = \frac{\partial N}{\partial x}$$

We can conclude that this is an exact equation. The solution is given by:

$$\psi(x,y) = C, \text{ where } C \text{ is a constant and } \frac{\partial \psi}{\partial x} = M \text{ and } \frac{\partial \psi}{\partial y} = N$$

To solve it we proceed as follow:

$$\psi(x,y) = \int \frac{\partial \psi}{\partial x}\partial x = \int (3x^2 - 2xy + 2)\partial x = x^3 - x^2 y + 2x + h(y)$$

Where $h(y)$ is a pure function of y. To find h we differentiate $\psi(x,y)$ w.r.t. y

$$\frac{\partial \psi}{y} = -x^2 + h'(y) = 6y^2 - x^2 + 3 \Longrightarrow h'(y) = 6y^2 + 3 \Longrightarrow \text{ (integrating both sides) } h(y) = 2y^3 + 3y$$

The general solution is given by:

$$\psi(x,y) = C = x^3 - x^2 y + 2x + 2y^3 + 3y$$

6. Consider the equation: $\dfrac{dy}{dx} = -\dfrac{ax - by}{bx - cy} \iff \dfrac{dy}{dx} = \dfrac{by - ax}{bx - cy} \iff (by - ax)dx + (-bx + cy)dy = 0$.

Let $M(x,y) = by - ax$ and $N(x,y) = -bx + cy$. Then, $\dfrac{\partial M}{\partial y} = b \neq -b = \dfrac{\partial N}{x}$. **Not** exact equation.

13. Consider the equation: $(2x - y)dx + (2y - x)dy = 0$. Let $M(x,y) = 2x - y$ and $N(x,y) = 2y - x$. Since:

$$\frac{\partial M}{\partial y} = -1 = \frac{\partial N}{\partial x}$$

We can conclude that this is an exact equation. The solution is given by:

$$\psi(x,y) = C, \text{ where } C \text{ is a constant and } \frac{\partial \psi}{\partial x} = M \text{ and } \frac{\partial \psi}{\partial y} = N$$

To solve it we proceed as follow:

$$\psi(x,y) = \int \frac{\partial \psi}{\partial x}\partial x = \int (2x - y)\partial x = x^2 - xy + h(y)$$

Where $h(y)$ is a pure function of y. To find h we differentiate $\psi(x,y)$ w.r.t. y

$$\frac{\partial \psi}{\partial y} = -x + h'(y) = 2y - x \Longrightarrow h'(y) = 2y \Longrightarrow \text{ (integrating both sides) } h(y) = y^2$$

The general solution is given by:

$$\psi(x,y) = C = x^2 - xy + +y^2$$

Solving for the initial condition $y(x = 1) = 3$, we get that $C = 1^2 - 3 + 9 = 7$. So, the particular solution is:

$$x^2 - xy + y^2 = 7$$

We can check that this solution works by differentiation implicitly:

$$D(x^2 - xy + y^2) = D(7) \Longrightarrow 2x - y + xy' + 2yy' = 0 \Longrightarrow y' = \frac{y - 2x}{2y - x}$$

Replacing for y'in our original equation:

$$(2x-y)+(2y-x)y' = (2x-y)+(2y-x)\frac{y-2x}{2y-x} = 2x-y+y-2x = 0 \quad \text{, so this is the solution}$$

Finally, we can write the solution explicitly as a function $y(x)$ by solving for the quadratic equation:

$$(1)y^2-(x)y+(x^2-7)=0 \iff y(x) = \frac{x \pm \sqrt{x^2-4(1)(x^2-7)}}{2\cdot 1} \iff y(x) = \frac{x \pm \sqrt{28-3x^2}}{2}$$

However, our final equation is that which contains the initial condition $(x_0, y_0) = (1, 3)$, i.e.:

$$y(x) = \frac{x+\sqrt{28-3x^2}}{2}$$

This solution is valid only if $28 - 3x^2 > 0 \iff x^2 < \dfrac{28}{3} \iff |x| < \dfrac{28}{3}$

14. Consider the equation: $(9x^2+y-1)dx - (4y-x)dy = 0 \iff (9x^2+y-1)dx + (x-4y)dy = 0$.
Let $M(x, y) = (9x^2 + y - 1)$ and $N(x, y) = (x - 4y)$. Since:

$$\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x}$$

We can conclude that this is an exact equation. The solution is given by:

$$\psi(x,y) = C, \text{ where } C \text{ is a constant and } \frac{\partial \psi}{\partial x} = M \text{ and } \frac{\partial \psi}{\partial y} = N$$

To solve it we proceed as follow:

$$\psi(x,y) = \int \frac{\partial \psi}{\partial x}\partial x = \int (9x^2+y-1)\partial x = 3x^3 + xy - x + h(y)$$

Where $h(y)$ is a pure function of y. To find h we differentiate $\psi(x, y)$ w.r.t. y

$$\frac{\partial \psi}{\partial y} = x + h'(y) = x - 4y \Longrightarrow h'(y) = -4y \Longrightarrow \text{ (integrating both sides) } h(y) = -2y^2$$

The general solution is given by:

$$\psi(x,y) = C = 3x^3 + xy - x - 2y^2$$

Solving for the initial condition $y(x = 1) = 0$, we get that $C = 3 + 0 - 1 - 0 = 2$. So, the particular solution is:

$$3x^3 + xy - x - 2y^2 = 2$$

We can check that this solution works by differentiation implicitly:

$$D(3x^3+xy-x-2y^2) = D(2) \Longrightarrow 9x^2 + y + xy' - 1 - 4yy' = 0 \Longrightarrow y' = \frac{1-y-9x^2}{x-4y}$$

Replacing for y'in our original equation:

$$(9x^2+y-1)+(x-4y)y' = (9x^2+y-1)+(x-4y)\frac{1-y-9x^2}{x-4y} = 9x^2+y-1+1-y-9x^2 = 0 \quad \text{, so this is the solution}$$

Finally, we can write the solution explicitly as a function $y(x)$ by solving for the quadratic equation:

$$(-2)y^2+(x)y+(3x^3-x-2) = 0 \iff y(x) = \frac{-x \pm \sqrt{x^2-4(-2)(3x^3-x-2)}}{2\cdot(-2)} \iff y(x) = \frac{x \pm \sqrt{24x^3+x^2-8x-16}}{4}$$

However, our final equation is that which contains the initial condition $(x_0, y_0) = (1, 0)$, i.e.:

$$y(x) = \frac{x-\sqrt{24x^3+x^2-8x-16}}{4}$$

This solution is valid only if $24x^3 + x^2 - 8x - 16 > 0$.

16. Consider the equation: $(ye^{2xy}+x)dx+(bxe^{2xy})dy=0$. Let $M(x,y)=ye^{2xy}+x$ and $N(x,y)=bxe^{2xy}$.

This equation is exact only if:

$$\frac{\partial M}{\partial y}=e^{2xy}+2xye^{2xy}=\frac{\partial N}{\partial x}=be^{2xy}+2bxye^{2xy}$$

We can conclude that this is an exact equation when $b=1$. So, let us solve the exact equation: $(ye^{2xy}+x)dx+(xe^{2xy})dy=0$. Let $M(x,y)=ye^{2xy}+x$ and $N(x,y)=xe^{2xy}$.
The solution is given by:

$$\psi(x,y)=C,\text{ where } C \text{ is a constant and } \frac{\partial\psi}{\partial x}=M \text{ and } \frac{\partial\psi}{\partial y}=N$$

To solve it we proceed as follow:

$$\psi(x,y)=\int\frac{\partial\psi}{\partial y}\partial y=\int(xe^{2xy})\partial y=\frac{e^{2xy}}{2}+h(x)$$

Where $h(x)$ is a pure function of x. To find h we differentiate $\psi(x,y)$ w.r.t. x

$$\frac{\partial\psi}{\partial x}=ye^{2xy}+x=ye^{2xy}+h'(x)\Longrightarrow h'(x)=x\Longrightarrow \text{ (integrating both sides) } h(x)=\frac{x^2}{2}$$

The general solution is given by:

$$\psi(x,y)=C=\frac{e^{2xy}}{2}+\frac{x^2}{2}\Longrightarrow e^{2xy}+x^2=C$$

We can check that this solution works by differentiation implicitly:

$$D(e^{2xy}+x^2)=D(C)\Longrightarrow e^{2xy}(y+xy')+x=0\Longrightarrow y'=\frac{-x-ye^{2xy}}{xe^{2xy}}$$

Replacing for y' in our original equation:

$$(ye^{2xy}+x)+(xe^{2xy})y'=(ye^{2xy}+x)+(xe^{2xy})\frac{-x-ye^{2xy}}{xe^{2xy}}=ye^{2xy}+x-x-ye^{2xy}=0 \quad \text{, so this is the solution}$$

19. The equation $x^2y^3+x(1+y^2)y'=0$ is not exact since, letting $M_1=x^2y^3$ and $N_1=x(1+y^2)$, it follows:

$$\frac{\partial M_1}{\partial y}3x^2y^2\neq\frac{\partial N_1}{\partial x}=1+y^2$$

But, the following equivalent equation is exact: $\frac{1}{xy^3}[x^2y^3+x(1+y^2)y'=0] \iff x+\left(\frac{1+y^2}{y^3}\right)y'=0$ Since, letting $M(x,y)=x$ and $N(x,y)=\frac{1+y^2}{y^3}$:

$$\frac{\partial M}{\partial y}=0=\frac{\partial N}{\partial x}$$

The solution is given by:

$$\psi(x,y)=C,\text{ where } C \text{ is a constant and } \frac{\partial\psi}{\partial x}=M \text{ and } \frac{\partial\psi}{\partial y}=N$$

To solve it we proceed as follow:

$$\psi(x,y)=\int\frac{\partial\psi}{\partial x}\partial x=\int x\partial x=\frac{x^2}{2}+h(y)$$

Where $h(y)$ is a pure function of y. To find h we differentiate $\psi(x,y)$ w.r.t. y

$$\frac{\partial\psi}{\partial y}=\left(\frac{1+y^2}{y^3}\right)=h'(y)\Longrightarrow h'(y)=\left(\frac{1+y^2}{y^3}\right)\Longrightarrow \text{ (integrating both sides) } h(y)=ln(|y|)-\frac{1}{2y^2}$$

The general solution is given by:

$$\psi(x,y) = C = \frac{x^2}{2} + ln(|y|) - \frac{1}{2y^2} \iff \text{ multiplying by 2 both sides... } \quad C_1 = x^2 + 2ln(|y|) - y^{-2}$$

We can check that this solution works by differentiation implicitly:

$$D(x^2 + 2ln(|y|) - y^{-2}) = D(C_1) \Longrightarrow 2x + \frac{2y'}{y^3} + \frac{2y'}{y} = 0 \Longrightarrow y' = \frac{-xy^3}{1+y^2}$$

Replacing for y' in our original equation:

$$x + \left(\frac{1+y^2}{y^3}\right) y' = x + \left(\frac{1+y^2}{y^3}\right) \frac{-xy^3}{1+y^2} = x - x = 0 \quad \text{, so this is the solution}$$

21. The equation $ydx + (2x - ye^y)dy = 0$ is not exact since, letting $M_1 = y$ and $N_1 = (2x - ye^y)$, it follows:

$$\frac{\partial M_1}{\partial y} = 1 \neq \frac{\partial N_1}{\partial x} = 2$$

But, the following equivalent equation is exact: $y[y + (2x - ye^y)y' = 0] \iff y^2 + (2xy - y^2e^y)y' = 0$ Since, letting $M(x,y) = y^2$ and $N(x,y) = (2xy - y^2e^y)$:

$$\frac{\partial M}{\partial y} = 2y = \frac{\partial N}{\partial x}$$

The solution is given by:

$$\psi(x,y) = C, \text{ where } C \text{ is a constant and } \frac{\partial \psi}{\partial x} = M \text{ and } \frac{\partial \psi}{\partial y} = N$$

To solve it we proceed as follow:

$$\psi(x,y) = \int \frac{\partial \psi}{\partial x} \partial x = \int y^2 \partial x = xy^2 + h(y)$$

Where $h(y)$ is a pure function of y. To find h we differentiate $\psi(x,y)$ w.r.t. y

$$\frac{\partial \psi}{\partial y} = 2xy - y^2e^y = 2xy + h'(y) \Longrightarrow h'(y) = -y^2e^y$$

To obtain $h(y)$ we integrate by parts setting $u = y^2 \Longrightarrow du = 2ydy; \quad v = e^y \Longrightarrow dv = e^y dy$

$$\begin{array}{rcl} \int y^2e^y dy & = & y^2e^y - 2\int ye^y \\ & = & y^2e^y - 2(e^y(y-1)) \\ & = & y^2e^y - 2ye^y + 2e^y \\ & = & e^y(y(y-2)+2) \\ & = & -h(y) \end{array}$$

Hence, $h(y) = -e^y(y(y-2)+2)$. The general solution is given by:

$$C = xy^2 - e^y(y(y-2)+2)$$

We can check that this solution works by differentiation implicitly:

$$D(xy^2 - e^y(y(y-2)+2)) = D(C) \Longrightarrow y^2 + 2xyy' - 2yy'e^y - y^2e^yy' + 2y'e^y + 2ye^yy' - 2e^yy' = 0 \Longrightarrow y' = \frac{-y^2}{2xy - y^2e^y}$$

Replacing for y' in our original equation:

$$y^2 + (2xy - y^2e^y)y' = y^2 + (2xy - y^2e^y)\frac{-y^2}{2xy - y^2e^y} = y^2 - y^2 = 0 \quad \text{, so this is the solution}$$

27. The equation $dx + (\frac{x}{y} - sin(y))dy = 0$ is not exact since, letting $M_1(x,y) = 1$ and $N_1(x,y) = \frac{x}{y} - sin(y)$:

$$\frac{\partial M_1}{\partial y} = 0 \neq \frac{\partial N_1}{\partial x} = \frac{1}{y}$$

We need to find the integrating factor. Assumer that $\mu = \mu(y)$, a pure function of y. Then, μ is given by:

$$\frac{u'}{u} = \frac{\frac{\partial N}{\partial x} - \frac{\partial N}{\partial y}}{M} = \frac{\frac{1}{y} - 0}{1} = \frac{1}{y} \Longrightarrow u' - \frac{1}{y}u = 0$$

This is a linear, 1st O.D.E. Solve by separating: $u = y$ (check: since $u' = 1$ and $u = y$ and hence, $u'/u = 1/y$).

Now, the equation: $y + (x - ysin(y))y' = 0$ is exact since, letting $M = y$ and $N = x - ysin(y)$, follows:

$$\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x}$$

The solution is given by:

$$\psi(x,y) = C, \text{ where } C \text{ is a constant and } \frac{\partial \psi}{\partial x} = M \text{ and } \frac{\partial \psi}{\partial y} = N$$

To solve it we proceed as follow:

$$\psi(x,y) = \int \frac{\partial \psi}{\partial x} \partial x = \int y \partial x = xy + h(y)$$

Where $h(y)$ is a pure function of y. To find h we differentiate $\psi(x,y)$ w.r.t. y

$$\frac{\partial \psi}{\partial y} = x - ysin(y) = x + h'(y) \Longrightarrow h'(y) = -ysin(y) \Longrightarrow \text{ (integrating both sides) } h(y) = ycos(y) - sin(y)$$

The general solution is given by:

$$\psi(x,y) = C = xy + ycos(y) - sin(y)$$

We can check that this solution works by differentiation implicitly:

$$D(xy + ycos(y) - sin(y)) = D(C) \Longrightarrow y + xy' - cos(y)y' + cos(y)y' - ysin(y)y' \Longrightarrow y' = \frac{-y}{x - ysin(y)}$$

Replacing for y' in our original equation:

$$y + (x - ysin(y))y' = y + (x - ysin(y))\frac{-y}{x - ysin(y)} = y - y = 0 \quad \text{, so this is the solution}$$

Section 2.7

1. The equation $y' = 3 + t - y$ with $y(0) = 1$ is a first order, linear equation. We solve this by integrating factor:

   (i) Rewrite the equation in the standard form $y' + y = 3 + t$

   (ii) Integrating factor: since $p(t) = 1$ we get $\mu(t) = e^{\int p(t)dt} = e^{\int 1dt} = e^t$

   (iii) Multiply both sides of the equation by the integrating factor: $e^t[y' + y] = e^t[3 + t]$

   (vi) Using product rule and implicit differentiation: $\frac{d}{dt}[e^t y] = e^t[3 + t]$

   (v) Integrate both sides: $\int \frac{d}{dt}[e^t y] = \int e^t[3 + t]dt \Longrightarrow e^t y = e^t(t + 2) + C$

The general solution is $y(t) = (t+2) + Ce^{-t}$.

Solving for the initial conditions: $y(0) = 1 = 2 + C \iff C = -1$. The particular solution is:

$$y(t) = (t+2) - e^{-t}$$

The following table is a comparison of Euler's method for different values of h and the exact solution.

t	Exact	$h = 0.1$	$h = 0.05$	$h = 0.025$
0.0	1.00000	1.00000	1.00000	1.00000
0.1	1.19516	1.20000	1.19750	1.19631
0.2	1.38127	1.39000	1.38549	1.38335
0.3	1.55918	1.57100	1.56491	1.56200
0.4	1.72968	1.74390	1.73658	1.73308

The best approximate solution from Euler's method is when $h = 0.025$.

4. The equation $y' = 3cos(t) - 2y$ with $y(0) = 0$ is a first order, linear equation. We solve this by integrating factor:

   (i) Rewrite the equation in the standard form $y' + 2y = 3cos(t)$

   (ii) Integrating factor: since $p(t) = 2$ we get $\mu(t) = e^{\int p(t)dt} = e^{\int 2dt} = e^{2t}$

   (iii) Multiply both sides of the equation by the integrating factor: $e^{2t}[y' + 2y] = 3e^{2t}cos(t)$

   (vi) Using product rule and implicit differentiation: $\dfrac{d}{dt}[e^{2t}y] = 3e^{2t}cos(t)$

   (v) Integrate both sides: $\int \dfrac{d}{dt}[e^{2t}y] = \int 3e^{2t}cos(t)dt \Longrightarrow e^t y = \dfrac{3}{5}e^{2t}(sin(t) + 2cos(t)) + C$

   The general solution is $y(t) = \dfrac{3}{5}(sin(t) + 2cos(t)) + Ce^{-2t}$.

   Solving for the initial conditions: $y(0) = \dfrac{6}{5} + C \iff C = -\dfrac{6}{5}$. The particular solution is:

   $$y(t) = \frac{3}{5}(sin(t) + 2cos(t)) - \frac{6}{5}e^{-2t}$$

   The following table is a comparison of Euler's method for different values of h and the exact solution.

t	Exact	$h = 0.1$	$h = 0.05$	$h = 0.025$
0.0	0.00000	0.00000	0.00000	0.00000
0.1	0.27143	0.30000	0.28481	0.27792
0.2	0.49090	0.53850	0.51334	0.50181
0.3	0.66514	0.72482	0.69345	0.67895
0.4	0.79973	0.86646	0.83157	0.81530

   The best approximate solution from Euler's method is when $h = 0.025$.

Section 3.1

9. Consider the equation $y'' + y' - 2y = 0$ with initial conditions: $y(0) = 1$ and $y'(0) = 1$.
   This is a linear, homogeneous, 2nd O.D.E. The solution is given by $y(t) = e^{rt}$. Then $y' = re^r$, $y'' = r^2e^{rt}$. So,

   $$\begin{array}{rcll} 0 & = & r^2e^{rt} + re^{rt} - 2e^{rt} & \text{Sub. for the solution} \\ & = & e^{rt}(r^2 + r - 2) & \text{Common factor } e^{rt} \\ & \iff & r^2 + r - 2 = 0 & \text{Since } e^{rt} \text{ is never zero} \\ & \iff & (r+2)(r-1) = 0 & \text{Factoring } r \end{array}$$

   Hence, the general solution is:

   $$y(t) = C_1e^{-2t} + C_2e^t$$

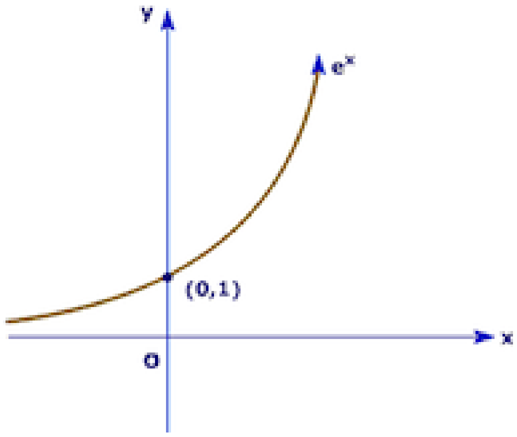
   Solving for the initial conditions:

   $$\begin{cases} y(0) = 1 = C_1e^{-2\cdot 0} + C_2e^0 = C_1 + C_2 \Longrightarrow & C_1 = 1 - C_2 \Longrightarrow C_1 = 1 - 1 = 0 \\ \\ y'(0) = 1 = -2C_1e^{-2\cdot 0} + C_2e^0 = -2C_1 + C_2 \Longrightarrow & 1 = -2 + 3C_2 \Longrightarrow C_2 = 1 \end{cases}$$

The particular solution is:

$$y(t) = e^t$$

We can check that this is the solution: $y(t) = y'(t) = y''(t) = e^t$, and hence $y'' + y' - 2y = e^t + e^t - 2e^t = 0$.
Graph of the solution:



Finally, $\lim_{t\to\infty} y(t) = \infty$, the solution goes to infinity very quick!.

10. Consider the equation $y'' + 4y' + 3y = 0$ with initial conditions: $y(0) = 2$ and $y'(0) = -1$.
This is a linear, homogeneous, 2nd O.D.E. The solution is given by $y(t) = e^{rt}$. Then $y' = re^r$, $y'' = r^2e^{rt}$.
So,

$$\begin{array}{rcll} 0 & = & r^2e^{rt} + 4re^{rt} + 3e^{rt} & \text{Sub. for the solution} \\ & = & e^{rt}(r^2 + 4r + 3) & \text{Common factor } e^{rt} \\ & \iff & r^2 + 4r + 3 = 0 & \text{Since } e^{rt} \text{ is never zero} \\ & \iff & (r+1)(r+3) = 0 & \text{Factoring } r \end{array}$$

Hence, the general solution is:

$$y(t) = C_1e^{-t} + C_2e^{-3t}$$

Solving for the initial conditions:

$$\begin{cases} y(0) = 2 = C_1e^0 + C_2e^0 = C_1 + C_2 \Longrightarrow C_1 = 1 - C_2 \Longrightarrow & C_1 = 2 + \frac{1}{2} = \frac{5}{2} \\ \\ y'(0) = -1 = -C_1e^{-t} - 3C_2e^{-3t} = -C_1 - 3C_2 \Longrightarrow 1 = C_1 + 3C_2 \Longrightarrow & 1 = -2 - C_2 + 3C_2 \Longrightarrow C_2 = -\frac{1}{2} \end{cases}$$

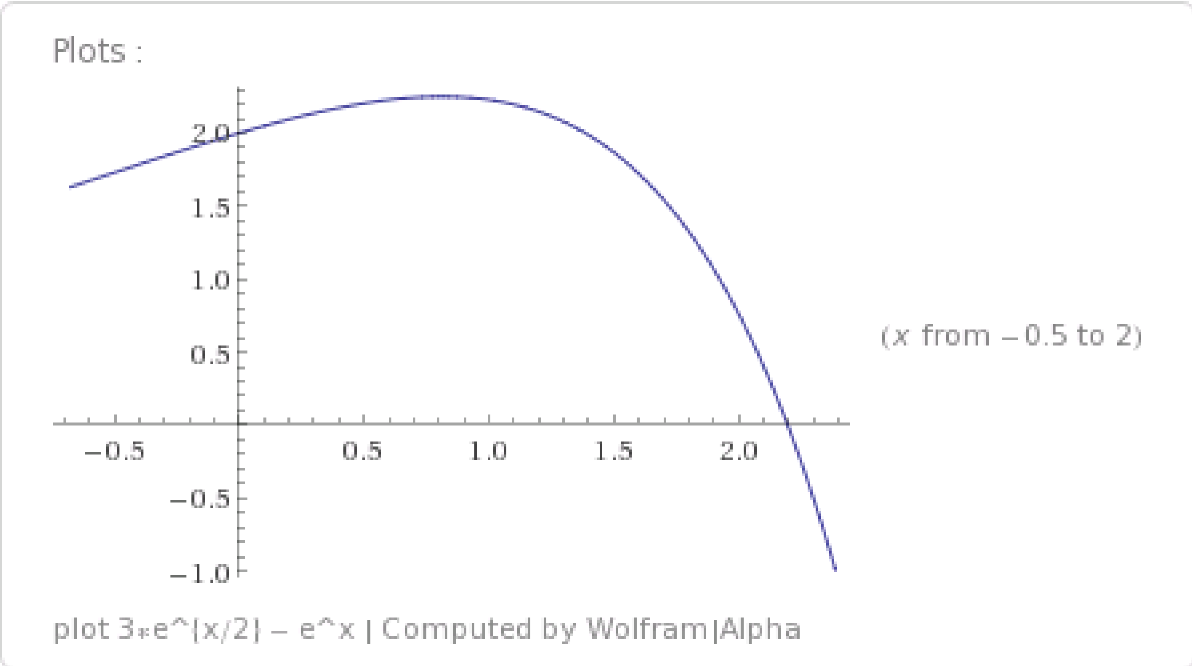
The particular solution is:

$$y(t) = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}$$

We can check that this is the solution: $y' = -\dfrac{5}{2}e^{-t} + \dfrac{3}{2}e^{-3t}$ and $y'' = \dfrac{5}{2}e^{-t} - \dfrac{9}{2}e^{-3t}$. Hence,

$$y''+4y'+3y = \frac{5}{2}e^{-t} - \frac{9}{2}e^{-3t} - \frac{20}{2}e^{-t} + \frac{12}{2}e^{-3t} + \frac{15}{2}e^{-t} - \frac{3}{2}e^{-3t} = e^{-t}\left(\frac{5}{2} - \frac{20}{2} + \frac{15}{2}\right) + e^{-3t}\left(\frac{12}{2} - \frac{9}{2} - \frac{3}{2}\right) = 0$$

Graph of the solution:



Finally, $\lim_{t\to\infty} y(t) = 0 - 0 = 0$, the solution converges to zero.

20. Consider the equation $2y'' - 3y' + y = 0$ with initial conditions: $y(0) = 2$ and $y'(0) = \frac{1}{2}$.
This is a linear, homogeneous, 2nd O.D.E. The solution is given by $y(t) = e^{rt}$. Then $y' = re^{r}$, $y'' = r^2 e^{rt}$. So,

$$\begin{array}{rcll} 0 & = & 2r^2e^{rt} - 3re^{rt} + e^{rt} & \text{Sub. for the solution} \\ & = & e^{rt}(2r^2 - 3r + 1) & \text{Common factor } e^{rt} \\ & \iff & 2r^2 - 3r + 1 = 0 & \text{Since } e^{rt} \text{ is never zero} \\ & \iff & (r-1)(r-\frac{1}{2}) = 0 & \text{Factoring } r \end{array}$$

Hence, the general solution is:

$$y(t) = C_1e^t + C_2e^{t/2}$$

Solving for the initial conditions:

$$\begin{cases} y(0) = 2 = C_1 + C_2 \Longrightarrow C_1 = 2 - C_2 & \Longrightarrow C_1 = -1 \\ \\ y'(0) = \frac{1}{2} = C_1 + \frac{1}{2}C_2 \iff 1 = 2C_1 + C_2 & \Longrightarrow C_2 = 3 \end{cases}$$

The particular solution is:

$$y(t) = 3e^{t/2} - e^t$$

We can check that this is the solution: $y' = \dfrac{3}{2}e^{t/2} - e^t$ and $y'' = \dfrac{3}{4}e^{t/2} - e^t$. Hence,

$$2y'' - 3y' + y = \frac{6}{4}e^{t/2} - 2e^t - \frac{9}{2}e^{t/2} + 3e^t + 3e^{t/2} - e^t = e^{t/2}\left(\frac{6}{4} - \frac{9}{2} + 3\right) + e^t\,(3 - 2 - 1) = 0$$

The solution is zero at t_0 if:

$$y(t_0) = 0 = 3e^{t_0/2} - e^{t_0} \Longrightarrow e^{t_0} = 3e^{t_0/2} \Longrightarrow t_0 = ln(3) + \frac{t_0}{2} \Longrightarrow t_0 = 2ln(3) \Longrightarrow t_0 = ln(9)$$

The solution is maximum at:

$$y'(t_0) = 0 = \frac{3}{2}e^{t_0/2} - e^{t_0} \Longrightarrow e^{t_0} = \frac{3}{2}e^{t_0/2} \Longrightarrow t_0 = ln(\frac{3}{2}) + \frac{t_0}{2} \Longrightarrow t_0 = 2ln(\frac{3}{2}) \Longrightarrow t_0 = ln(\frac{9}{4})$$

Since, $y''(ln(\dfrac{9}{4})) = \dfrac{3}{4}e^{ln(\frac{9}{4})/2} - e^{ln(\frac{9}{4})} = \dfrac{3}{4} \cdot \dfrac{16}{81} - \dfrac{9}{4} < 0$, the point $t_0 = ln(\dfrac{9}{4})$ is a local maximum.
The value of the maximum is $y(ln(\dfrac{9}{4})) = 3e^{ln(\frac{9}{4})/2} - e^{ln(\frac{9}{4})} = \dfrac{9}{4}$