Midterm Exam 2

Math-M311 Spring 2011

March 10, 2011

Answer the questions in the spaces provided on the question sheets, being sure to provide full justification for your solutions. If you run out of room for an answer, continue on the back of the page. Your exam should have 5 pages. Please check to make sure your exam is complete.

Name: _____

1. (25 points)

(a) Determine and sketch the domain of the function

$$f(x, y) = \sqrt{x + y} \ \ln(x^2 + y^2 - 1).$$

(b) Find the following limit, if it exists, or show that the limit does not exist:

$$\lim_{(x,y) \to (0,0)} \frac{6x^2 y^3}{\sqrt{x^2 + 2y^2}}.$$

(c) Determine the set of all points where the following function is continuous:

$$f(x, y) = \begin{cases} \frac{xy}{x^2+3y^2}, & \text{if } (x, y) \neq (0,0); \\ 0, & \text{if } (x, y) = (0,0). \end{cases}$$

(Don't forget to explain what happens away from $(0,0)$.)

(d) If $u = 2x^3 + yz^2$, where $x = pr\cos(\theta)$, $y = r^2\sin(\theta)$ and $z = p + r$, find $\frac{\partial u}{\partial p}$ when $p = 2$, $r = 3$, $\theta = \frac{\pi}{2}$.

2. (25 points) Consider the function $f(x, y) = ye^{xy}$.

    (a) Find the directional derivative of f at the point $(1, 2)$ in the direction of the vector $\vec{v} = \langle 3, -4 \rangle$.

    (b) Find the maximum rate of change of f at the point $(1, 2)$, and determine the direction in which it occurs.

3. (25 points) Consider the function $f(x,y) = \sqrt{y + \cos^2(x)}$.

   (a) Find an equation for the tangent plane to the surface $z = f(x,y)$ at the point $(0,0)$.

   (b) Use part (a) to approximate the value of $f(-0.05, 0.1) = \sqrt{0.1 + \cos^2(-0.05)}$.

   (c) Show the surface $z = f(x,y)$ intersects the surface $2z^2 + (x+4)y - 2z\cos(x) = 0$ perpendicularly at the point $(0,0,1)$. (This means that the surfaces intersect at this point and that their tangent planes are perpendicular at this point of intersection.)

4. (20 points) Find and classify the critical points of the function $f(x, y) = x^3 - 12xy + 8y^3$.