CSCI B609- Midterm I

February 28, 2013
Duration: 90 Minutes

Instructions

1. Answer each question on a new page in your blue books.
2. This is an open book, open note exam.
3. Make sure you write LEGIBLY and give enough explanation whenever it is due.
4. No electronic devices are allowed.

Q1. (5 pts.) Use Lipski's algorithm to construct a universal sequence for $X = \{1, 2, 3, 4, 5, 6\}$.

Q2. (5 pts.) Prove that every bipartite graph G with l edges has a matching of size at least $l/\Delta(G)$, where $\Delta(G)$ is the maximum degree of a vertex in G.
Hint: Use König-Egerváry Theorem for bipartite graphs.

Q3. (5 pts.) Use Pascal Triangle identity to show that

$$\sum_{i=0}^{r} \binom{n+i-1}{i} = \binom{n+r}{r}.$$

Q4. (5 pts.) Consider the graph Q_3 where the vertices are binary strings of size 3 and there is an edge between u and v iff u and v are Hamming distance 1 apart.

- Show that Q_3 is bipartite: Give the bipartition and the edges.
- Use the M-augmenting path algorithm we discussed in class to find a maximum matching in Q_3.

Q5. (5 pts.) Show that the number of pairs (A, B) of distinct subsets of $\{1, 2, \ldots, n\}$ with $A \subset B$ is $3^n - 2^n$.
Hint: Consider the expression $\sum_{k=0}^{n} \binom{n}{k} (2^k - 1)$.

Q6. (5 pts.) (*Extra Credit*)
Show that for any positive integer n, there is a multiple of n that contains only the digits 7 or 0.
Hint: Consider the values modulo n of all the numbers a_i of the form $77\cdots7$, with i sevens for $i = 1, 2, \ldots, n+1$.