

Name: Enrique Areyan

Score: _____/100

EXAM 1

M464

Prof. Lyons

Spring 2014

Explain all answers. If you use a formula or method we covered in class, you do not need to re-derive that formula here. Just state it clearly. If you find yourself doing long calculations, then you are doing the problem a hard way or incorrectly.

1. (25 points) A Markov chain has the transition probability matrix

$$P := \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0 & 1 \end{pmatrix} \end{matrix}.$$

For your convenience, here are the powers P^2, P^3, P^4 of P in case you wish to use any of them:

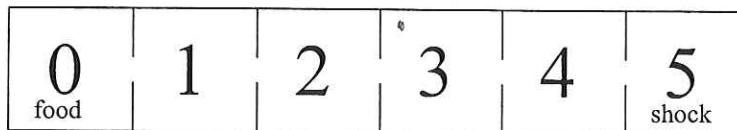
$$\begin{pmatrix} 0.4 & 0.22 & 0.38 \\ 0.22 & 0.29 & 0.49 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0.284 & 0.19 & 0.526 \\ 0.19 & 0.189 & 0.621 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0.2084 & 0.1518 & 0.6398 \\ 0.1518 & 0.1325 & 0.7157 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) Which states are absorbing?

(b) Let T be the first time that the chain is in an absorbing state. The chain starts in state $X_0 = 0$. What is the probability that $X_3 = 0$ given that $T > 3$? You may leave your answer as a fraction.

(c) What is the probability that the last non-absorbing state visited is 0? (Here, the chain starts in state $X_0 = 0$, but we do *not* condition that $T > 3$.)

2. (15 points) A cute lab rat is put into compartment 3 of the following linear maze:



Assume that the poor rat is equally likely to move right or left at each step. What is the probability that the rat finds the food in compartment 0 before getting unpleasantly shocked in compartment 5?

TURN OVER

3. (20 points) Five balls are distributed between two urns, labeled A and B. Each period, an urn is selected at random (probability $1/2$ each), and if the selected urn is not empty, a ball from that urn is removed and placed into the other urn. If the selected urn is empty, then no balls are moved.

- (a) In the long run, what fraction of time is urn A empty?
- (b) Is the Markov chain you used to solve part (a) regular?

4. (20 points) From purchase to purchase of a product, a particular customer switches brands among A, B, and C according to a Markov chain whose transition probability matrix is

$$\begin{array}{c} A \quad B \quad C \\ \begin{matrix} A \\ B \\ C \end{matrix} \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.7 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{pmatrix}. \end{array}$$

In order to answer the question, “In the long run, what fraction of time does this customer purchase brand A?”, write down what equations you would solve and why (what theorems or methods are you using?), but do not actually carry out the arithmetic or algebra. Make sure you say how you get the answer from your method (such as, “The answer is the value of x from the following equations . . .”).

5. (20 points) Determine the communicating classes and periods for each state of the Markov chain whose transition probability matrix is

$$\begin{array}{c} 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}. \end{array}$$