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M451/551 Quiz 3

February 3, Prof. Connell

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1. A five-year \$10,000 bond with a 10% coupon rate costs \$10,000 and pays its holder \$500 every six months for five years, starting at the end of the sixth month, with a final additional payment of \$10,000 made at the end of those ten payments. Write down its present value if the (nominal yearly) interest rate is 6%. Assume the compounding is monthly. (You do not need to simplify your expression.)

STREAM:

PAYMENT	-10,000	500	500	500	500	500	500	500	500	500	500 + 10,000
YEAR	0	1/2	1	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5

$$PV(STREAM) = -10{,}000 + 500(\alpha)^6 + 500(\alpha^2)^6 + 500(\alpha^3)^6 + \cdots + 500(\alpha^{10})^6 + 10{,}000(\alpha^{10})^6,$$

where $\alpha = \frac{1}{1+\frac{0.06}{12}} = \frac{1}{1+0.005} = \frac{1}{1.005}$

Hence, the Present value is:

$$PV(STREAM) = -10{,}000 + \sum_{i=1}^{10} 500(1.005)^{-6i} + 10{,}000(1.005)^{-60}$$

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(Problem #2 is on the other side.)

2. Show that the yield curve $\bar{r}(t)$ is a nondecreasing function of t if and only if $P(\alpha t) \geq (P(t))^{\alpha}$ for all $0 \leq \alpha \leq 1$, ~~$t \leq 0$~~ $t \geq 0$. Recall $P(t) = \exp\left(-\int_0^t r(s)ds\right)$.

($\Rightarrow$) Suppose $\bar{r}(t)$ is a nondecreasing function of t.
this means: $\forall t_1, t_2$: If $t_1 \geq t_2$ then $\bar{r}(t_1) \geq \bar{r}(t_2)$

Now, note that $\bar{r}(t) = \frac{1}{t}\int_0^t r(s)ds$. Note that since $\alpha \in [0,1]$; $t \geq \alpha t$. It follows:

$\bar{r}(t) \geq \bar{r}(\alpha t)$, Moreover, e^{x} is an increasing function.

Hence, $e^{\bar{r}(t)} \geq e^{\bar{r}(\alpha t)}$. But we can write

$$e^{\bar{r}(t)} = e^{\frac{1}{t}\int_0^t r(s)ds} = \left[e^{-\frac{1}{\alpha t}\int_0^t r(s)ds}\right]^{\alpha} [P(t)]^{\alpha}$$

$$e^{\bar{r}(\alpha t)} = e^{\frac{1}{\alpha t}\int_0^{\alpha t} r(s)ds} = e^{-t\int_0^{\alpha t} r(s)ds} = P(\alpha t)$$

Therefore, $P(\alpha t) \geq (P(t))^{\alpha}$, which follows because e^{-x} is decreasing for $x > 0$.

($\Leftarrow$) Note that this direction follows just from reading the previous proof from bottom to top.

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